LECTURE 21

Project appraisal

Escalating cashflows, accumulated profit, and the IRR

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From securities to projects

$$1 + i \;=\; (1 + j)(1 + i_r), \qquad i_r \;=\; \frac{i - j}{1 + j}$$

Say asecurity is bought, or a *project* is undertaken. How do we appraise one?

Lecture 20 for fixed money payments under constant inflation j.

Today

1. Escalating cashflows
2. Accumulated profit
3. Net present value versus accumulated profit
4. Profitability and the internal rate of return

Warm-up

EXERCISE 21.1

A venture requires €1,000.00 *now and returns* €400.00, €500.00 *and* €600.00 *at the ends of the next three years. The investor can borrow or lend at* 10%*. What is the net present value?*

Warm-up

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$$\text{NPV}(0.10) = -1000 + \frac{400}{1.1} + \frac{500}{1.1^2} + \frac{600}{1.1^3} = -1000 + 1227.65 = €227.65.$$

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POSITIVE, SO THE VENTURE IS WORTH DOING AT 10%.

But those figures are in *today's* prices. If the venture is undertaken, the money that actually changes hands in three years' time will have different buying power.

SECTION 1

Escalating cashflows

Escalation vs inflation

We have used the term inflation without defining it, as it is coequally understood. But there is a less common but related term, and we will now give their definitions, as defining them together helps to keep them apart.

DEFINITION 21.2

Inflation is the rise in the general price level of a broad basket of goods and services in an economy. It measures the overall decline in purchasing power.

Escalation is the rise in the price of a specific item or cost. This increase can be higher or lower than general inflation because it is driven by factors unique to that item, such as supply and demand, new regulations, or resource scarcity.

> WE WILL NOT DIFFERENTIATE BETWEEN THEM
>
> For the purposes of this module, we will treat them mostly interchangeably.

The constant escalation assumption

The case in which all items of cashflow are subject to the same rate of **escalation** j per unit time is of special interest. We find or estimate c_t^e and $\rho^e(t)$, the net cashflow and net rate of cashflow allowing for escalation, by

$$c_t^e \;=\; (1+j)^t c_t, \qquad\qquad \rho^e(t) \;=\; (1+j)^t \rho(t),$$

where c_t and $\rho(t)$ are estimates *without* any allowance for inflation.

> KEY IDEA
>
> So c_t is the cashflow in today's prices, and c_t^e is what will actually change hands at time t once prices have risen. The whole project is scaled up, date by date.

The net present value with escalation

With allowance for inflation at rate j, the net present value at rate of interest i is

$$\begin{aligned}\mathrm{NPV}_j(i) &= \sum_t c_t(1+j)^t(1+i)^{-t} + \int_0^\infty \rho(t)(1+j)^t(1+i)^{-t}\,\mathrm{d}t \\ &= \sum_t c_t(1+i_0)^{-t} + \int_0^\infty \rho(t)(1+i_0)^{-t}\,\mathrm{d}t,\end{aligned}$$

since $(1+j)^t(1+i)^{-t} = \left(\frac{1+i}{1+j}\right)^{-t}$, where

$$1+i_0 = \frac{1+i}{1+j} \qquad \text{that is} \qquad i_0 = \frac{i-j}{1+j}.$$

The same algebra as last lecture

KEY IDEA

The cashflows was *fixed* in money terms in Lecture 20 and then deflated, obtaining $i_r = \frac{i-j}{1+j}$. Here, the cashflows is *escalated* and then discounted, and we get the same $i_0 = \frac{i-j}{1+j}$.

In both cases $(1 + j)$ and $(1 + i)$ meet as a ratio, and the effect of inflation is absorbed into a single adjusted rate.

If j is not too large the approximation $i_0 \approx i - j$ is sometimes used; for $i = 10\%$ and $j = 5\%$ it gives 5% against the exact 4.762%. This is a useful sanity-check.

Why this matters

KEY IDEA

Projects apparently unprofitable when interest rates are high may become highly profitable when even a modest allowance is made for inflation, as escalating income financed at i behaves like *level* income financed at the lower rate i_0.

EXAMPLE 21.3

In the warm-up venture with cashflow $-1000, +400, +500, +600$ at $t = 0, 1, 2, 3$, and $i = 10\%$, if we escalate at $j = 5\%$, then $i_0 = \frac{0.05}{1.05} = 4.762\%$. This give us

$$\text{NPV}_j(0.10) \;=\; -1000 + \frac{400}{1.04762} + \frac{500}{1.04762^2} + \frac{600}{1.04762^3} \;=\; €359.24,$$

against the €227.65 of the warm-up. More than 50% increase in profitability.

SECTION 2

Accumulated profit

What a project looks like

To assess a project we need a cashflow model.

OUTGO $(-)$	The outlay. Usualy an initial payment, possibly with subsequent outlays.
INCOME $(+)$	Returns, interest, dividends, proceeds of further investment.
UNCERTAINTY	In both the timing and the amount of the cashflows. It can be in the form of further investment, taxes, and so on.
RISKS	Delays, default, force majeure.

> ASIDE
>
> Cashflow estimation and risk assessment require experience, judgement, a set of assumptions (optimistic, realistic, pessimistic) and often a statistical model. The arithmetic below is the easy part. The real challenge is getting the numbers.

Accumulated profit

DEFINITION 21.4 · Accumulated profit

The **accumulated profit** is the accumulated value of the net cashflows at the time of the last payment. For a project ending at time T, at effective rate i per unit time,

$$A(T) = \sum_t (1+i)^{T-t} c_t + \int_0^T (1+i)^{T-t} \rho(t)\,\mathrm{d}t$$

It is immediate from the definition that $A(T) = (1+i)^T \,\mathrm{NPV}(\mathcal{C})$, but more on this later.

Appraisal: Choose the project with the highest accumulated profit.

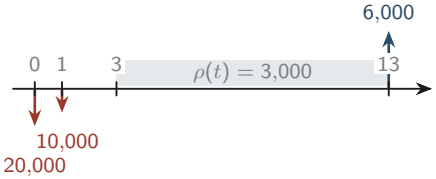
APPRAISAL NEEDS A *DEFINITIVE* END DATE

Disadvantage: We must know when the project ends, and we need $T < \infty$. A perpetual cashflow has no defined accumulated profit.

Example

EXAMPLE 21.5

An investor estimates that a new venture will require an initial outlay of €20,000.00 and a further outlay of €10,000.00 after one year. Beginning in three years, an inflow of €3,000.00 p.a. payable continuously for ten years is expected, and a final inflow of €6,000.00 at the end of the project in thirteen years. Determine the accumulated profit at an effective rate of 1.2% per annum.



Example: the cashflows

The net cashflows are

$$c_0 = -20{,}000, \qquad c_1 = -10{,}000, \qquad c_{13} = +6{,}000,$$

together with $\rho(t) = +3{,}000$ for $3 \le t \le 13$. Hence

$$A(13) = -20{,}000(1.012)^{13} - 10{,}000(1.012)^{12} + 6{,}000 + \int_3^{13} 3{,}000\,(1.012)^{13-t}\,\mathrm{d}t.$$

Note that we take $13 - t$, not the more familiar $t - 13$, since we are not discounting to time 0, but accumulating to time 13.

KEY IDEA

The integral is an accumulated continuous annuity. Substituting $u = 13 - t$ turns it into $\int_0^{10} 3000(1.012)^u\,\mathrm{d}u = 3{,}000\,\bar{s}_{\overline{10}|}$, ten years of continuous payment accumulated to the moment they stop.

Example: the arithmetic

From Lecture 7, $\bar{s}_{\overline{n}|} = \dfrac{(1+i)^n - 1}{\delta}$ with $\delta = \ln(1.012)$, so

$$3{,}000\,\bar{s}_{\overline{10}|} \;=\; 3{,}000\left(\frac{(1.012)^{10}-1}{\ln(1.012)}\right) \;=\; 3{,}000 \times 10.620868 \;=\; €\,31{,}862.60,$$

while the three discrete flows accumulate to

$$-20{,}000(1.167741) - 10{,}000(1.153895) + 6{,}000 \;=\; -€\,28{,}893.77.$$

Therefore $A(13) = -28{,}893.77 + 31{,}862.60 = €\,2{,}968.83 > 0$.

SECTION 3

Net present value versus accumulated profit

Two dates, one cashflow

Recall that the net present value of a project ending at $T \in (0, \infty]$ at effective rate i is

$$\mathsf{NPV}(i) \;=\; \sum_t \mathsf{v}^t c_t + \int_0^T \mathsf{v}^t \rho(t)\,\mathrm{d}t,$$

where c_t is the net flow at time t (inflow - outflow), and $\rho(t)$ likewise the rate of net flow.

$A(T)$ VALUES AT THE *END*; NPV(i) VALUES AT THE *START*

Every summand of $A(T)$ is $(1+i)^T$ times the corresponding summand of NPV(i), so

$$\boxed{A(T) \;=\; (1+i)^T \cdot \mathsf{NPV}(\mathcal{C})}$$

for $T < \infty$.

The two appraisals are equivalent

KEY IDEA

This is Lecture 4's transfer principle applied to a cashflow and thus, for $(1+i)^T > 0$, $A(T)$ and NPV($\mathcal{C}$) have the same sign. The highest accumulated profit and the highest NPV are the same project. In particular, profitable means NPV(i) > 0.

THE DATES MUST MATCH

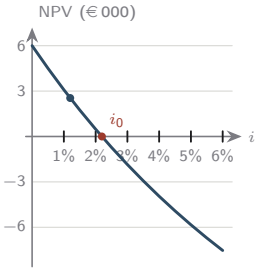
Of course, comparing $A(T_1)$ with $A(T_2)$ for $T_1 \neq T_2$ compares two different moments, and can rank the projects wrongly.

The example revisited

For the investor's venture,

$$\text{NPV}(0.012) \;=\; \frac{A(13)}{(1.012)^{13}} \;=\; \frac{2{,}968.83}{1.167741} \;=\; €2{,}542.37 \;>\; 0,$$

so the project is profitable at 1.2%, as the positive accumulated profit already told us.



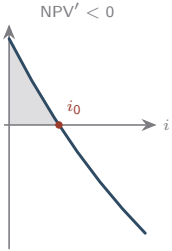
Here NPV is *decreasing*, and $\text{NPV}(i) > 0$ exactly for $i < i_0$, where i_0 is the yield.

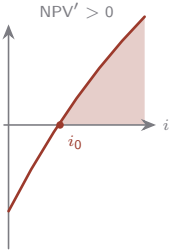
SECTION 4

Profitability and the internal rate of return

The general rule

Let i be the rate at which the investor may lend or borrow. Assume moreover that the yield i_0 exists and NPV(i) changes sign at $i = i_0$:





The crossing may go either way, so we must specify the slope. The project is profitable if and only if $\text{NPV}'(i_0) < 0$ and $i < i_0$, or $\text{NPV}'(i_0) > 0$ and $i > i_0$.

Here, $\text{NPV}'(i_0)$ denotes the derivative with respect to time.

The internal rate of return

Internal rate of return (IRR), or **yield to redemption**, is the terminology of economics and accountancy for the yield i_0, if it exists. It is the perceived rate of return if the project is *not* financed by a loan. That is, if financed internally.

KEY IDEA

Both cases combine into one inequality. The project is profitable if and only if

$$(i - i_0) \cdot \left(\frac{\mathrm{d}}{\mathrm{d}i} \mathrm{NPV} \right) (i_0) \; > \; 0.$$

Appraise by testing it at the given rate i. If negative, the project is set to loose money.

Reading the rule

EXAMPLE 21.6

For the investor's venture $i_0 = 2.197\%$, and NPV is decreasing, so $\mathrm{NPV}'(i_0) < 0$.

Rate available	$i - i_0$	NPV(i)	Verdict
$i = 0.5\%$	< 0	$+€4{,}503.00$	Profitable
$i = 1.2\%$	< 0	$+€2{,}542.00$	Profitable
$i = 3\%$	> 0	$-€1{,}854.00$	Loses money

KEY IDEA

The venture earns 2.197% internally. Borrow more cheaply and you keep the difference; borrow more dearly and the loan costs more than the venture makes.

Lecture 21 on one slide

ONE PROJECT, VALUED AT EITHER END

$$A(T) \;=\; (1+i)^T \,\mathrm{NPV}(i), \quad \text{profitable} \iff \mathrm{NPV}(i) > 0$$

$$\text{Escalation at } j: \;\; i_0 \;=\; \tfrac{i-j}{1+j}, \quad \text{profitable} \iff (i - i_0)\,\mathrm{NPV}'(i_0) > 0.$$

Escalating cashflows discounted at i behave like level ones discounted at i_0. The IRR is a $property of the project alone; whether it's worth doing depends on the rate the investor gets.$

Exercises

EXERCISE 21.7

A project costs €5,000.00 *now and returns* €2,000.00 *at the end of each of three years, at* $i = 6\%$. *Find* NPV(0.06) *and* $A(3)$, *and check that* $A(3) = (1.06)^3\,\mathrm{NPV}(0.06)$.

EXERCISE 21.8

The same project's cashflows now escalate at $j = 3\%$, *still financed at* 6%. *Find* i_0 *and the new net present value, and compare with the approximation* $i_0 \approx i - j$.

EXERCISE 21.9

A project has NPV *increasing in* i, *with yield* $i_0 = 8\%$. *Is it profitable at* 5%*? At* 11%*?*

Solutions are in the backup slides.

END OF LECTURE 21

One project, appraised. Next: comparing two of them, and noting that higher yields don't always win.

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Solution 1

At 6%, $a_{\overline{3}|} = 2.673012$, so

$$\text{NPV}(0.06) \;=\; -5000 + 2000\,(2.673012) \;=\; €346.02.$$

Accumulating each flow to time 3 instead,

$$A(3) \;=\; -5000(1.06)^3 + 2000\left((1.06)^2 + 1.06 + 1\right) \;=\; -5955.08 + 6367.20 \;=\; €412.12.$$

Checking the identity, $346.02 \times (1.06)^3 = 346.02 \times 1.191016 = €412.12$, as claimed. Both are positive, so the project is profitable. The numbers were only needed for this.

Solution 2

Escalation at $j = 3\%$ with financing at $i = 6\%$ gives

$$i_0 \;=\; \frac{0.06 - 0.03}{1.03} \;=\; 0.029126,$$

and the escalated project is then valued as a *level* one at that rate:

$$\mathrm{NPV}_j(0.06) \;=\; -5000 + 2000\, a_{\overline{3}|} \text{ at } 2.9126\% \;=\; -5000 + 2000\,(2.833370) \;=\; €666.74.$$

Escalation nearly doubles the value, from €346.02.

The approximation $i_0 \approx i - j = 0.03$ is high by 0.00087. Small, but it discounts every term slightly too hard and costs about €9.52 here. The error grows with the term of the project.

Solution 3

With $\text{NPV}'(i_0) > 0$, the rule

$$(i - i_0) \cdot \text{NPV}'(i_0) \; > \; 0$$

requires $i > i_0$.

So at 5% the project is *not* profitable, since $(0.05 - 0.08)$ is negative and the product with a positive slope is negative. At 11% it *is*, since $(0.11 - 0.08)\,\text{NPV}'(i_0) > 0$.

This is the reverse of the usual pattern, and is exactly why the slope must appear in the rule. It arises when the project takes money in first and pays it out later. This is a borrowing rather than a lending, for which a higher interest rate is good news.