LECTURE 20

Inflation and real yields

What an investor is paid, and what it can buy

Dr Ilia Pirashvili
Department of Mathematics, University of Galway
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An assumption nobody has questioned



Thus far, we moved money through time and compared the amounts. But a euro at time t does not buy what a euro at time 0 buys. Every yield so far has been a *money* yield.

Today

1. Real and money yields
2. Constant inflation
3. Payments linked to inflation
4. Index-linked bonds

Warm-up

EXERCISE 20.1

For an investment of 100 *at time* 0 *an investor receives* 120 *at time* 1*. Prices over the year rise by* 5%*. By what percentage has the investor's purchasing power increased?*

Warm-up

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Goods that cost 1 at time 0 cost 1.05 at time 1. As such 120 at time 1 buys what $\frac{120}{1.05} = 114.286$ would have bought at time 0. Against a cost of 100, the increase in purchasing power is

$$\frac{114.286}{100} - 1 \;=\; 14.286\%.$$

> ASIDE
>
> The face value the investor earned is 20%. But $20\% - 5\% = 15\%$ is *not* the answer.

SECTION 1

Real and money yields

What a real rate of interest is

DEFINITION 20.2 · Real rate of interest

The **real rate of interest** of a transaction is the rate of interest after allowing for the effect of inflation on a payment series.

The effect of inflation is that a unit of money at time 0 has different purchasing power from a unit at other times. We find the real rate by first adjusting all payments for inflation, so that they are expressed in units of purchasing power *at the same date*.

> BEHIND THE JARGON
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> The ordinary yield, with no adjustment, is the **money yield**, or the **nominal yield**. Everything before now has been about nominal yields.

Using an inflation index

Where the rates of inflation are *known*, that is, during completed (past) transactions, we may adjust payments by reference to a relevant inflation index $Q(t)$.

Time t	0	1	2	3
Cashflow c_t	-100	8	8	108
Index $Q(t)$	150	156	166	175

The money yield on this transaction is 8%. It is a three-year par bond with an 8% coupon. To convert to time-0 money values, divide the payment at time t by the proportional increase in the index:

$$\frac{8}{Q(1)/Q(0)} = \frac{8}{156/150} = 7.6923.$$

The real cashflows and the real yield

Time t	0	1	2	3
Nominal c_t	-100	8	8	108
$Q(0)/Q(t)$	1	150/156	150/166	150/175
In time-0 money	-100	7.6923	7.2289	92.5714

From this we obtain a yield equation for the real yield i_r,

$$-100 + 7.6923\,\mathsf{v}_{i_r} + 7.2289\,\mathsf{v}_{i_r}^2 + 92.5714\,\mathsf{v}_{i_r}^3 \;=\; 0,$$

and solving by interpolation, exactly as in Lecture 10, gives $i_r = 2.63\%$.

So an 8% bond returned barely 2.6% in purchasing power.

The general equation, and the base date

In general, for cashflows $c_{t_1}, \ldots, c_{t_n}$ with index values $Q(0), Q(t_1), \ldots, Q(t_n)$, the real yield equation in time-0 money units is

$$\sum_{k=1}^{n} c_{t_k} \frac{Q(0)}{Q(t_k)} \mathsf{v}_{i_r}^{t_k} \;=\; 0.$$

- Prices rose by a factor $\frac{Q(t_k)}{Q(0)}$, so c_{t_k} is worth $c_{t_k}\frac{Q(0)}{Q(t_k)}$ in time-0 money.
- $Q(t_k)$ *deflates* the payment back to time 0; v_{i_r} then *discounts* it at the real rate.

KEY IDEA

Multiplying through by $1/Q(0)$ gives $\sum_k \frac{c_{t_k}}{Q(t_k)} \mathsf{v}_{i_r}^{t_k} = 0$, in which $Q(0)$ has vanished/ The answer does not depend on which date the payments are expressed in.

SECTION 2

Constant inflation

The real yield equation

Assume inflation is at a constant rate j per annum, so a cashflow c at time t is worth $c(1+j)^{-t}$ in time-0 terms. The real rate i_r and the money yield i are expressable by the following equations:

$$\sum_{k=1}^{n} c_{t_k} \mathsf{v}_j^{t_k} \mathsf{v}_{i_r}^{t_k} = 0 \qquad \text{and} \qquad \sum_{k=1}^{n} c_{t_k} \mathsf{v}_i^{t_k} = 0.$$

KEY IDEA

We have the coefficients, the same exponents, and thus, the two equations will have the same solutions precisely when

$$\mathsf{v}_i = \mathsf{v}_j \mathsf{v}_{i_r}.$$

The relationship between the three rates

From $\mathsf{v}_i = \mathsf{v}_j \mathsf{v}_{i_r}$ we get $\frac{1}{1+i} = \frac{1}{(1+j)(1+i_r)}$, that is $1 + i = (1 + j)(1 + i_r)$. Solving for i_r,

$$\boxed{i_r = \frac{i - j}{1 + j}} \qquad \text{and conversely} \qquad \boxed{i = i_r + j(1 + i_r)}$$

For the warm-up, $i = 0.20$ and $j = 0.05$ give $i_r = \frac{0.15}{1.05} = 0.142857$, which we found there by hand.

> CAUTION
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> Note $i_r \neq i - j$. Subtracting inflation is only a first approximation, and it *overstates* the real return here by 0.71 percentage points.

SECTION 3

Payments linked to inflation

Contracts that adjust for inflation

Some contracts specify that the cashflows *will* be adjusted to allow for future inflation, usually in terms of a given index. The actual cashflows are then unknown until the index values are.

The contract specifies a nominal amount c_t at time t. If the index at the base date is $Q(0)$ and the relevant value for the time-t payment is $Q(t)$, the actual cashflow at time t is

$$c_t \frac{Q(t)}{Q(0)} \qquad \text{(the payment adjusted } \textit{upwards} \text{ for inflation).}$$

Contrast the last section, where the payments were fixed in money terms and we deflated them. Here they are indexed *up*, and we shall then deflate them back down.

When the two indices agree

If the real yield i_r is calculated by reference to the *same* index as is used to inflate the cashflows, then i_r solves

$$\sum_{k=1}^{n} \underbrace{\left(c_{t_k} \frac{Q(t_k)}{Q(0)} \right)}_{\text{payment inflated}} \times \underbrace{\frac{Q(0)}{Q(t_k)}}_{\text{deflated back}} \mathsf{v}_{i_r}^{t_k} = 0.$$

The two index factors are reciprocals, so they cancel and $\sum_k c_{t_k} \mathsf{v}_{i_r}^{t_k} = 0$.

> KEY IDEA
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> The yield equation may be solved using the *nominal* amounts. The indexation is exactly undone by the deflation, and the nominal amounts are already the real ones.

An illustration, and a warning

For example, if the coupons of the government bond of Lecture 15 were linked to an index Q, the inflation-adjusted half-yearly coupon at time t would be

$$19.13\,\frac{Q(t)}{Q(0)},$$

which in time-0 money has real value 19.13 as before.

> CAUTION
>
> It is **not** always the case that the index used to inflate the cashflows is the one used to compute the real yield. Moreover, as the next section shows, even the same index will fail to cancel if it is applied with a *time lag*.

SECTION 4

Index-linked bonds

Index-linked bonds and the time lag

An **index-linked bond** has interest and redemption payments linked to an inflation index. Computing a price or a yield requires an estimate of the index, since future values are unknown.

> WHY THERE IS A LAG
>
> A time lag is needed from the date of the index value used, to allow for its publication and for the coupon amount to be calculated in advance. *There is no inflation protection during the lag period.*

With a lag s, a coupon of 19.13 at time t becomes $19.13\,\frac{Q(t-s)}{Q(0-s)}$, of real value $19.13\,\frac{Q(t-s)}{Q(0-s)}\cdot\frac{Q(0)}{Q(t)}$. We now have four index values, and they no longer cancel.

The lag matters

Take a lag of one year, so $s = 1$, and write out the real value of the coupon at time t:

$$3.5\,\frac{Q(t-1)}{Q(-1)}\cdot\frac{Q(0)}{Q(t)} \;=\; 3.5\,\underbrace{\frac{Q(0)}{Q(-1)}}_{1+j_{\text{base}}}\cdot\underbrace{\frac{Q(t-1)}{Q(t)}}_{1/(1+j_t)} \;=\; 3.5\,\frac{1+j_{\text{base}}}{1+j_t},$$

where j_t is the inflation rate over the year ending at t.

KEY IDEA

The lag protects the coupon against the inflation of the *base* year, not the current one. While inflation is steady the two nearly cancel; when it jumps, the real coupon falls.

Example 1: the data

EXAMPLE 20.3

Consider a 3.5% coupon stock issued on February 2014 and redeemed in February 2019. Coupons are annual and linked to an inflation index with a one-year time lag. The base month for indexation is February 2013.

Year	2013	2014	2015	2016	2017	2018	2019
Index value	540	562	584	607	632	657	788
Inflation %		4.07	3.91	3.94	4.12	3.96	19.94

Take February 2014 as time 0, so the coupons fall at $t = 1, \ldots, 5$, the base month is $t = -1$, and the lag is $s = 1$. In other words, exactly the above example/case.

Example 1: two coupons in detail

The coupon at time t is $3.50\,\dfrac{Q(t-1)}{Q(-1)}$, and its real value in February 2014 terms is that times $\dfrac{Q(0)}{Q(t)}$.

EXAMPLE 20.4

At $t = 2$ (February 2016):

$$3.5 \times \frac{Q(1)}{Q(-1)} \;=\; 3.5 \times \frac{584}{540} \;=\; 3.79, \qquad \text{real} \;=\; 3.79 \times \frac{562}{607} \;=\; 3.51.$$

At $t = 3$ (February 2017): $3.5 \times \frac{607}{540} = 3.93$, with real value $3.93 \times \frac{562}{632} = 3.49$.

Example 1: the whole table

Year	2015	2016	2017	2018	2019
Coupon per €100.00	3.64	3.79	3.93	4.10	4.26
Real value, Feb 2014 terms	3.51	3.50	3.50	3.50	3.04

With no lag the real value would be exactly 3.50 every year. With it, the value stays within a cent of 3.50 for four years — and then collapses.

KEY IDEA

Looking at the inflation index data explains this. Inflation jumped to 19.94% in 2019 against a base year of 4.07%. Hence, the real coupon is $3.5 \times \frac{1.0407}{1.1994} = 3.04$.

Example 2: the question

EXAMPLE 20.5

A four-year index-linked security, issued on 1 April 2015, pays nominal coupons of 8% every two years in arrears and is redeemed at 110%. Coupons and capital are subject to inflation by reference to an index Q six months before payment.

(a) Find the effective rate of interest i per annum earned over the life of the security, if the price is €100.00 per €100.00 nominal on 1 April 2015.

(b) Find the real rate of return i_r per annum.

Date	1/10/14	1/4/15	1/10/16	1/4/17	1/4/18	1/10/18	1/4/19
Q	601	631	662	695	730	788	877
Time	−0.5	0	1.5	2	3	3.5	4

(a) The money yield

The lifetime is $[0,4]$ with $0 \leftrightarrow 1/4/15$, and the lag is $s = 0.5$. A cashflow c_t at time t adjusted for inflation is $c_t \frac{Q(t-s)}{Q(0-s)}$. Coupons of 8 fall at $t = 2, 4$ and the redemption of 110 at $t = 4$, so per 100 nominal,

$$100 \;=\; 8\mathrm{v}_i^2 \frac{Q(1.5)}{Q(-0.5)} + 118\mathrm{v}_i^4 \frac{Q(3.5)}{Q(-0.5)} \;=\; 8\mathrm{v}_i^2 \frac{662}{601} + 118\mathrm{v}_i^4 \frac{788}{601},$$

that is $100 = 8.81\mathrm{v}_i^2 + 154.72\mathrm{v}_i^4$.

> CAUTION
>
> Note which index values appear. The payment at $t = 2$ uses the index six months *earlier*, at $t = 1.5$, and the base is $Q(-0.5)$, not $Q(0)$.

(a) Solving the quadratic

We must solve $154.72\mathrm{v}_i^4 + 8.81\mathrm{v}_i^2 - 100 = 0$. Put $x = \mathrm{v}_i^2 > 0$, then

$$154.72x^2 + 8.81x - 100 \;=\; 0,$$

with roots $x = 0.775979$ and $x = -0.8329$. We discard the negative root since $x > 0$. So $\mathrm{v}_i^2 = 0.775979$ and

$$i \;=\; \frac{1}{\mathrm{v}_i} - 1 \;=\; \frac{1}{\sqrt{0.775979}} - 1 \;=\; 13.52\%.$$

KEY IDEA

The substitution $x = \mathrm{v}^2$ turns a quartic into a quadratic. It is only available here because both payments fall at *even* times. It is the same as in Lecture 11 with $x = \frac{2}{3}\mathrm{v}^2$.

(b) The real rate of return

The real time-0 value of a time-t cashflow c_t is the inflation adjustment followed by the deflation:

$$\left(c_t \times \frac{Q(t-s)}{Q(0-s)}\right) \times \frac{Q(0)}{Q(t)}.$$

Thus in real terms, per 100 nominal,

$$100 \;=\; 8\mathrm{v}_{i_r}^2 \,\frac{(662)(631)}{(601)(695)} + 118\mathrm{v}_{i_r}^4 \,\frac{(788)(631)}{(601)(877)} \;=\; 8\mathrm{v}_{i_r}^2 + 111.32\mathrm{v}_{i_r}^4.$$

The first coefficient came out at almost exactly 8: over that particular two-year stretch the lag happened to cost the investor almost nothing.

(b) Solving, and what it means

Solving $111.32\mathrm{v}_{i_r}^4 + 8\mathrm{v}_{i_r}^2 - 100 = 0$ for $\mathrm{v}_{i_r}^2$ and again discarding the negative root,

$$\mathrm{v}_{i_r}^2 \;=\; 0.912541 \qquad \Longrightarrow \qquad i_r \;=\; \frac{1}{\sqrt{0.912541}} - 1 \;=\; 4.68\%.$$

KEY IDEA

The investor earned 13.52% in money terms but only 4.68% in purchasing power. The two are consistent, as Q rose from 631 to 877 over four years, an average of 8.58% per annum, and $(1.0468)(1.0858) - 1 = 13.66\%$. This i close to 13.52%, the small gap being the effect of the lag.

Lecture 20 on one slide

DEFLATE FIRST, THEN DISCOUNT

$$\sum_k c_{t_k} \frac{Q(0)}{Q(t_k)} \mathsf{v}_{ir}^{t_k} = 0, \qquad 1+i = (1+j)(1+i_r), \qquad i_r = \frac{i-j}{1+j}.$$

The base date cancels, so the real yield does not depend on it. Indexing to the same index used for deflating makes the two cancel, and the nominal amounts *are* the real ones. However, a time lag breaks that cancellation, protecting the payment against base-period inflation rather than current.

Exercises

EXERCISE 20.6

*€*500.00 *returns €*650.00 *after two years, with inflation at* 4%*. Find the money and real yields, and check* $i_r = (i - j)/(1 + j)$.

EXERCISE 20.7

A bond has $Q(0) = 200$, $Q(3) = 260$, $Q(-1) = 190$, $Q(2) = 245$ *and a coupon of nominal €*4.00*. Find the coupon at* $t = 3$ *and its real value, with no lag and with a one-year lag.*

EXERCISE 20.8

In Example 1 the real coupons stayed near 3.50*. What would push one above it?*

Solutions are in the backup slides.

END OF LECTURE 20

Money yields say what you were paid; real yields say what it could buy. Next: the same arithmetic pointed forwards, at a project whose costs and revenues both escalate.

ilia.pirashvili@universityofgalway.ac.ie

Solution 1

The money yield solves $500(1+i)^2 = 650$, so

$$i \;=\; \left(\frac{650}{500}\right)^{1/2} - 1 \;=\; 14.0175\%.$$

For the real yield, deflate the receipt: €650.00 at time 2 is worth $650/(1.04)^2 = €600.96$ in time-0 money, so

$$i_r \;=\; \left(\frac{600.96}{500}\right)^{1/2} - 1 \;=\; 9.6323\%.$$

Checking against the formula,

$$\frac{i-j}{1+j} \;=\; \frac{0.140175 - 0.04}{1.04} \;=\; 0.096323,$$

which agrees. Note again that $i - j = 10.02\%$ would have overstated it.

Solution 2

No lag. The actual coupon is $4 \times \frac{Q(3)}{Q(0)} = 4 \times \frac{260}{200} = €5.20$, and its real value is

$$5.20 \times \frac{Q(0)}{Q(3)} \;=\; 5.20 \times \frac{200}{260} \;=\; €4.00,$$

the two factors being reciprocals, so they cancel exactly.

One-year lag. Now the coupon is $4 \times \frac{Q(2)}{Q(-1)} = 4 \times \frac{245}{190} = €5.16$, with real value

$$5.1579 \times \frac{200}{260} \;=\; €3.97.$$

It is no longer €4.00 because the indexation used $Q(2)/Q(-1)$ while the deflation used $Q(0)/Q(3)$: different periods, so no cancellation. The shortfall is the inflation of the year from $t = 2$ to $t = 3$, against which the coupon was never protected.

Solution 3

The real value $3.5\,\dfrac{1+j_{\text{base}}}{1+j_t}$ exceeds 3.50 exactly when

$$\frac{1+j_{\text{base}}}{1+j_t} > 1 \qquad \Longleftrightarrow \qquad j_t < j_{\text{base}},$$

that is, when inflation in the current year is *lower* than it was in the base year.

So falling inflation benefits the holder of a lagged index-linked bond, and rising inflation hurts them. This is the opposite of what such a bond is usually assumed to do. The protection is real but delayed, and the delay is exactly one lag period.