LECTURE 14

Schedules in general

A moving rate, a whole table, and payments between the years

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Three assumptions still standing

$$\underbrace{b_t = iL_{t-1}}_{\text{Interest repayed}}, \qquad \underbrace{f_t = L_{t-1} - L_t}_{\text{Capital repaied}}$$

$$\underbrace{L_t \;=\; L_{t-1}(1+i) - X_t}_{\text{Total loan remaining}}$$

Everything so far has assumed one rate for the whole term, and one payment per period. We will drop both assumptions today.

Today

1. When the rate is not constant
2. The schedule
3. More frequent repayments
4. Fractional terms

Warm-up

EXERCISE 14.1

A loan of €1,000.00 *is repaid by two annual instalments of* €400.00 *and a final payment. The rate charged is* 5% *in the first year and* 6% *in the second. Find the loan outstanding after each of the two instalments.*

The rate moves, so no closed form applies. However, the recursion of Lecture 12 can still be used, as it never asked for a constant rate. Roll forward, subtract, repeat.

Warm-up

EXERCISE 14.1

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Add the interest of that term to the loan, then subtract the payment, at eash step:

$$L_1 \;=\; L_0(1+i_1) - X_1 \;=\; 1000(1.05) - 400 \;=\; €650.00,$$

$$L_2 \;=\; L_1(1+i_2) - X_2 \;=\; 650(1.06) - 400 \;=\; €289.00.$$

Warm-up

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$$L_1 = €650.00 \qquad L_2 = €289.00.$$

ASIDE

The final payment must therefore be $289.00\,(1 + i_3)$, whatever i_3 will be. This is why a lender reserving the right to change the rate can only quote the schedule one year at a time.

Note that we used no annuities or assumptions that instalments are equal.

SECTION 1

When the rate is not constant

Dropping the constant rate

Neither the prospective nor the retrospective method depends on the interest rate being constant. Suppose the rate charged from time $r-1$ to time r is i_r.



KEY IDEA

The only thing that changes is that instead of $(i+i)^n$, we now take $(1+i_1)\cdots(1+i_n)$. This is the principle of consistency in action.

Prospectively, with varying rates

At time $t+1$, the capital repaid is $L_t - L_{t+1}$ and the interest is $i_{t+1}L_t$, so

$$X_{t+1} \;=\; (1+i_{t+1})L_t - L_{t+1}, \qquad \text{i.e.} \qquad L_t \;=\; \frac{L_{t+1}+X_{t+1}}{1+i_{t+1}}.$$

Substituting repeatedly until $L_n = 0$:

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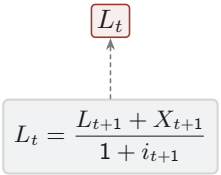

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Substituting repeatedly until $L_n = 0$:

$$L_t = \frac{X_{t+1}}{1+i_{t+1}} + \frac{L_{t+1}}{1+i_{t+1}}$$

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Substituting repeatedly until $L_n = 0$:

$$L_t = \frac{X_{t+1}}{1+i_{t+1}} + \frac{L_{t+1}}{1+i_{t+1}}$$

$$L_{t+1} = \frac{L_{t+2}+X_{t+2}}{1+i_{t+2}}$$

Prospectively, with varying rates

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Substituting repeatedly until $L_n = 0$:

$$L_t = \frac{X_{t+1}}{1+i_{t+1}} + \frac{L_{t+2}+X_{t+2}}{(1+i_{t+1})(1+i_{t+2})}$$

Prospectively, with varying rates

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Prospectively, with varying rates

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Substituting repeatedly until $L_n = 0$:

$$L_t = \frac{X_{t+1}}{1+i_{t+1}} + \frac{X_{t+2}}{(1+i_{t+1})(1+i_{t+2})} + \cdots + \frac{X_{t+k}}{(1+i_{t+1})\cdots(1+i_{t+k})} + \frac{L_{t+k}}{(1+i_{t+1})\cdots(1+i_{t+k})}$$

$$L_{t+k} = \frac{L_{t+k+1}+X_{t+k+1}}{1+i_{t+k+1}}$$

Here, $k = n - t$

Prospectively, with varying rates

At time $t+1$, the capital repaid is $L_t - L_{t+1}$ and the interest is $i_{t+1}L_t$, so

$$X_{t+1} \;=\; (1+i_{t+1})L_t - L_{t+1}, \qquad \text{i.e.} \qquad L_t \;=\; \frac{L_{t+1}+X_{t+1}}{1+i_{t+1}}.$$

Substituting repeatedly until $L_n = 0$:

$$L_t = \frac{X_{t+1}}{1+i_{t+1}} + \frac{X_{t+2}}{(1+i_{t+1})(1+i_{t+2})} + \cdots + \frac{X_n}{(1+i_{t+1})\cdots(1+i_n)} + \frac{\cancel{L_n}}{(1+i_{t+1})\cdots(1+i_n)}$$

$L_n = 0$

Prospectively, with varying rates

At time $t+1$, the capital repaid is $L_t - L_{t+1}$ and the interest is $i_{t+1}L_t$, so

$$X_{t+1} \;=\; (1+i_{t+1})L_t - L_{t+1}, \qquad \text{i.e.} \qquad L_t \;=\; \frac{L_{t+1}+X_{t+1}}{1+i_{t+1}}.$$

Substituting repeatedly until $L_n = 0$:

$$L_t = \boxed{\frac{X_{t+1}}{1+i_{t+1}} + \frac{X_{t+2}}{(1+i_{t+1})(1+i_{t+2})} + \cdots + \frac{X_n}{(1+i_{t+1})\cdots(1+i_n)}}$$

Prospectively, with varying rates

At time $t+1$, the capital repaid is $L_t - L_{t+1}$ and the interest is $i_{t+1}L_t$, so

$$X_{t+1} \;=\; (1+i_{t+1})L_t - L_{t+1}, \qquad \text{i.e.} \qquad L_t \;=\; \frac{L_{t+1}+X_{t+1}}{1+i_{t+1}}.$$

Substituting repeatedly until $L_n = 0$:

$$L_t = \boxed{X_{t+1}\mathsf{v}_{t+1} + X_{t+2}\,(\mathsf{v}_{t+1}\mathsf{v}_{t+2}) + \cdots + X_n\,(\mathsf{v}_{t+1}\ldots\mathsf{v}_n)}$$

Each instalment is discounted by the rates of exactly the years it travels back through.

Retrospectively, with varying rates

At time $t \geq 1$ the interest due is $b_t = i_t L_{t-1}$, so $f_t = X_t - i_t L_{t-1}$ and $L_t = L_{t-1}(1 + i_t) - X_t$. Working back to time 0,

$$\begin{aligned} L_t \;=\; & L_0(1+i_1)(1+i_2)\cdots(1+i_t) \\ & - \big(X_1(1+i_2)\cdots(1+i_t) + X_2(1+i_3)\cdots(1+i_t) + \cdots + X_{t-1}(1+i_t) + X_t\big). \end{aligned}$$

> **CAUTION**
>
> Note which rates appear: X_1, paid at time 1, is accumulated by $i_2, \ldots, i_t$ but *not* by i_1, as it was not in the fund during the first year. The last instalment X_t carries no factor at all.

SECTION 2

The schedule

The general schedule

The loan payments can be set out as a table, or **schedule**.

Year from - to	Loan o/s at r	Instalment at $r+1$	Interest due at $r+1$	Capital repaid at $r+1$	Loan o/s at $r+1$
0–1	L_0	X_1	iL_0	$X_1 - iL_0$	$L_1 = L_0 - (X_1 - iL_0)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
r–r+1	L_r	X_{r+1}	iL_r	$X_{r+1} - iL_r$	$L_{r+1} = L_r - (X_{r+1} - iL_r)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n–1–n	L_{n-1}	X_n	iL_{n-1}	$X_n - iL_{n-1}$	$L_n = 0$

"o/s" — outstanding

Each row uses only the row above it, so the whole schedule is a recursion. This is the kind of thing people love to do on spreadsheets.

The schedule for a level annuity

In the particular case when the interest i is fixed and we have a level anuity $a_{\overline{n}|}$ of size 1, the schedule becomes

Payment	Interest content	Capital repaid	Loan o/s after payment
1	$ia_{\overline{n}|} = 1 - \mathsf{v}^n$	v^n	$a_{\overline{n}|} - \mathsf{v}^n = a_{\overline{n-1}|}$
2	$ia_{\overline{n-1}|} = 1 - \mathsf{v}^{n-1}$	v^{n-1}	$a_{\overline{n-1}|} - \mathsf{v}^{n-1} = a_{\overline{n-2}|}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$ia_{\overline{n-t+1}|} = 1 - \mathsf{v}^{n-t+1}$	v^{n-t+1}	$a_{\overline{n-t+1}|} - \mathsf{v}^{n-t+1} = a_{\overline{n-t}|}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$ia_{\overline{1}|} = 1 - \mathsf{v}$	v	$a_{\overline{1}|} - \mathsf{v} = 0$

The right-hand column is the identity $a_{\overline{m}|} - \mathsf{v}^m = a_{\overline{m-1}|}$. To see this, observe that dropping the last term of $\mathsf{v} + \cdots + \mathsf{v}^m$ leaves the annuity one period shorter.

Scaling the schedule

KEY IDEA

If an amount L_0 is lent against n repayments of $L_0/a_{\overline{n}|}$, the monetary amounts in the lender's schedule are those of the table above multiplied by the constant factor

$$\frac{L_0}{a_{\overline{n}|}}.$$

So there is really only *one* level-annuity schedule, and every actual loan is a rescaling of it.

That is why the crossover month in Lecture 13 did not depend on the €10,000.00.

SECTION 3

More frequent repayments

No new principles

Most loans are repaid quarterly, monthly or weekly. No new principles are involved where payments are made more frequently than annually, but care is needed in computing the interest due at any instalment date.

> KEY IDEA
>
> If the rate of interest used is effective over the same time unit as the frequency of the instalments, the calculations proceed exactly as before, with the time unit redefined accordingly.

That is precisely what we did in Lecture 13. The rate was 1% per month and the payments monthly. Thus, "one month" became the unit and nothing else changed. What follows is the case where the rate is quoted *annually* but the payments are m-thly.

The equation of value

With repayments X_t at times $t = \frac{1}{m}, \frac{2}{m}, \ldots, \frac{mn-1}{m}, n$,

$$L_0 = X_{\frac{1}{m}} \mathsf{v}^{\frac{1}{m}} + X_{\frac{2}{m}} \mathsf{v}^{\frac{2}{m}} + \cdots + X_{\frac{mn-1}{m}} \mathsf{v}^{\frac{mn-1}{m}} + X_n \mathsf{v}^n.$$



Nothing really changes and we can again calculate it in two ways: At any repayment date the loan outstanding is the value of what remains, or the accumulated loan minus the accumulated repayments.

Both directions, unchanged

PROSPECTIVE

$$L_t \;=\; X_{t+1/m}\mathsf{v}^{1/m} + X_{t+2/m}\mathsf{v}^{2/m} + \cdots + X_n\mathsf{v}^{n-t}.$$

RETROSPECTIVE

$$L_t \;=\; L_0(1+i)^t - \big(X_{1/m}(1+i)^{\,t-1/m} + \cdots + X_{t-1/m}(1+i)^{1/m} + X_t\big).$$

The exponent on v is always the time from t to the payment, measured in whole units. An instalment $\frac{k}{m}$ of a unit after t is discounted by $\mathsf{v}^{\,k/m}$, and one $\frac{k}{m}$ of a unit before it is accumulated by $(1+i)^{\,k/m}$.

The interest due in one m-thly period

Given an annual effective rate i, the effective rate over a period $\frac{1}{m}$ is

$$(1+i)^{1/m} - 1 \;=\; \frac{i^{(m)}}{m},$$

see $i^{(m)}$ in Lecture 1. Thus, with capital outstanding L_t at a repayment date t,

$$b_{t+1/m} \;=\; \frac{i^{(m)}}{m}\, L_t, \qquad f_{t+1/m} \;=\; X_{t+1/m} - \frac{i^{(m)}}{m}\, L_t.$$

CAUTION

Not $\frac{i}{m}$. A twelfth of 6% is 0.5%, but the monthly compounding of 6% is 0.4868%.

This is Lecture 1's nominal-versus-effective distinction once again.

A half-yearly schedule

EXAMPLE 14.2

A loan of €1,000.00 is repayable by a level annuity payable half-yearly in arrears for three years, at 15% per annum effective. Construct the lender's schedule.

A half-yearly schedule

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The effective rate over a half-year is

$$\frac{i^{(2)}}{2} = (1.15)^{1/2} - 1 = 0.0723805,$$

so the interest due at the end of each half-year is 7.23805% of the loan outstanding at its start. Six payments of X at that effective rate give

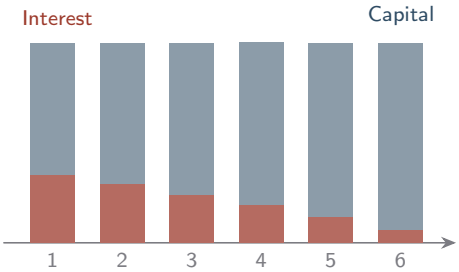
$$X\, a_{\overline{6}|} = 1000, \qquad X = \frac{1000 \times 0.0723805}{1 - (1.0723805)^{-6}} = €211.34.$$

The lender's schedule

n	Loan o/s at start of nth half-year	Interest due at end	Capital repaid at end	Loan o/s at end of nth half-year
		$0.0723805 \times$ Col 2	$211.34 -$ Col 3	Col 2 $-$ Col 4
1	1000.00	72.38	138.96	861.04
2	861.04	62.32	149.02	712.02
3	712.02	51.54	159.80	552.22
4	552.22	39.97	171.37	380.85
5	380.85	27.57	183.77	197.08
6	197.08	14.26	197.08	0.00
		268.04	1000.00	

Every row is one application of the recursion. The capital column sums to the loan and the balance runs exactly to zero. You should always do these two checks on a schedule.

Reading the schedule



Interest falls from €72.38 to €14.26. The capital rises from €138.96 to €197.08, by a factor 1.0723805 each half-year. This is Lecture 13's geometric progression, with the half-year as the period.

Total paid $6 \times 211.34 = €1{,}268.04$, of which €1,000.00 is capital and €268.04 interest.

SECTION 4

Fractional terms

A gap in the notation

In that example the annuity could also have been valued in *years*, as $2X\, a^{(2)}_{\overline{3}|}$. But $a^{(m)}_{\overline{n}|}$ was defined in Lecture 8 only for positive integers n. For certain non-integral n, the symbol has an obvious interpretation, and for others not so clear.

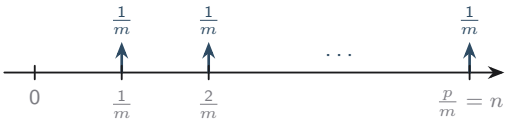
- $a^{(4)}_{\overline{23.5}|}$ ought to be the present value of an annuity of 1 per annum payable quarterly in arrears for 23.5 years. This is a total of 94 quarterly payments of 0.25.
- $a^{(2)}_{\overline{23.25}|}$ has no obvious meaning. 23.25 years is 46.5 half-years, and there is no such thing as half a payment.

That is, we ask if nm is a whole number.

> A NOTE ON FRACTIONAL TERMS
>
> Some writers define $a^{(p)}_{\overline{n}|} := a^{(p)}_{\overline{n_1}|} + f\mathsf{v}^n$ where $n = n_1 + f$ with n_1 an intiger multiple of $1/p$ and $0 < f < 1/p$. In that definition, we get $a^{(2)}_{\overline{23.75}|} = a^{(2)}_{\overline{23.5}|} + 1/4\mathsf{v}^{23.75}$.

The definition

Suppose n is an integer multiple of $\frac{1}{m}$, say $n = p/m$ with p an integer. We define $a^{(m)}_{\overline{n}|}$ to be the value at time 0 of p payments of equal size $\frac{1}{m}$, at times $\frac{1}{m}, \frac{2}{m}, \ldots, \frac{p}{m}$.



If $i = 0$ then $a^{(m)}_{\overline{n}|} = n$; the interesting case is $i > 0$.

The formula is unchanged

For $i > 0$, summing the geometric progression exactly as in Lecture 8 gives

$$a_{\overline{n}|}^{(m)} = \sum_{k=1}^{p} \frac{\mathrm{v}^{k/m}}{m} = \frac{\mathrm{v}^{1/m}\left(1-\mathrm{v}^{p/m}\right)}{m\left(1-\mathrm{v}^{1/m}\right)} = \frac{1-\mathrm{v}^{n}}{m\left(\mathrm{v}^{-1/m}-1\right)} = \frac{1-\mathrm{v}^{n}}{i^{(m)}},$$

using $p/m = n$ and $\mathrm{v}^{-1/m} = (1+i)^{1/m}$.

> KEY IDEA
>
> We get the exact same formula as nothing in its derivation depended on n being a whole number. We only needed for the payments to arrive at intervals of $\frac{1}{m}$ and stop at n. The *definition* has just been extended.

Changing the unit of time

Work in a new unit equal to $\frac{1}{m}$ of the old, at the equivalent effective rate $\frac{i^{(m)}}{m}$ per new unit:

$$\boxed{a^{(m)}_{\overline{n}|} \text{ at rate } i} = \boxed{\frac{1}{m}\, a_{\overline{nm}|} \text{ at rate } \frac{i^{(m)}}{m}}$$

This is why the two routes to the half-yearly instalment agreed. $a^{(2)}_{\overline{3}|}$ at 15% is $\frac{1}{2}a_{\overline{6}|}$ at 7.23805%.

> CAUTION
>
> If n is not an integer multiple of $\frac{1}{m}$ there is no universally recognised definition of $a^{(m)}_{\overline{n}|}$. See "a note on fractional terms" above.

The other three symbols

Whenever $n = p/m$, the symbols $\ddot{a}^{(m)}_{\overline{n}|}$, $s^{(m)}_{\overline{n}|}$ and $\ddot{s}^{(m)}_{\overline{n}|}$ are values at different dates of one annuity certain: p payments of $\frac{1}{m}$ at intervals of $\frac{1}{m}$. So, exactly as in Lecture 8,

$$\ddot{a}^{(m)}_{\overline{n}|} \;=\; (1+i)^{1/m} a^{(m)}_{\overline{n}|} \;=\; \frac{1-\mathrm{v}^n}{d^{(m)}},$$

$$s^{(m)}_{\overline{n}|} \;=\; (1+i)^n a^{(m)}_{\overline{n}|} \;=\; \frac{(1+i)^n - 1}{i^{(m)}}, \qquad \ddot{s}^{(m)}_{\overline{n}|} \;=\; \frac{(1+i)^n - 1}{d^{(m)}}.$$

RECAP

As before, the numerator just says where we value (start of finish) and the denominator depends on when the first payment is made. This is Lecture 9's table all over, now valid for fractional terms too.

A note on consumer credit: APR

As already mentioned in Lecture 1, there are various ways to display an interest rate: annually, weekly, daily, continuously, effectively, as a discount factor etc. Thus "1%0" looses it's meaning for consumers who are not used to this, and can lead to very deceptive practices. Loans also may have additional operating costs.

> APR
>
> To ensure that consumers can make informed judgements about the interest rates charged, lenders are required (in most circumstances) to give information about the effective rate of interest charged. In the UK this is in the form of the Annual Percentage Rate of charge, or APR, which is defined as the effective annual rate of interest, rounded to the nearer 1/10th of 1%.

Lecture 14 on one slide

LOANS FOR ANY RATE, AND ANNUITIES AT (SOME) FRACTIONAL INTERVALS

$$L_t = L_{t-1}(1+i_t) - X_t \qquad \text{(varying rate gives a product, not a power)}$$

$$b_{t+1/m} = \frac{i^{(m)}}{m} L_t, \qquad a^{(m)}_{\overline{n}|} = \frac{1-\mathsf{v}^n}{i^{(m)}} \text{ whenever } nm \in \mathbb{N}.$$

Choose the unit of time so that the rate in hand is effective, and the m-thly schedule is the annual one with the clock relabelled. Every level-annuity schedule is a rescaling of a single table.

Exercises

EXERCISE 14.3

A loan of €5,000.00 *is repayable by a level annuity payable quarterly in arrears for two years at* 12% *per annum effective. Find the instalment and the first two lines of the schedule.*

EXERCISE 14.4

For which of $a^{(2)}_{\overline{23.25}|}$, $a^{(4)}_{\overline{23.25}|}$, $a^{(12)}_{\overline{7.25}|}$, $a^{(4)}_{\overline{7.4}|}$ *is the symbol defined? Give a reason in each case.*

EXERCISE 14.5

A loan is repaid quarterly at an effective annual rate of 8%*. Find the effective quarterly rate and* $i^{(4)}$*, and the interest due in a quarter on a balance of* €20,000.00*.*

Solutions are in the backup slides.

END OF LECTURE 14

The machinery is finished. Next it is pointed at something new: a bond, and the price an investor should pay for one.

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Solution 1

Take the quarter as the unit, so that the rate in hand becomes effective:

$$(1.12)^{1/4} - 1 \;=\; 0.0287373 \;=\; \frac{i^{(4)}}{4}, \qquad n \;=\; 8.$$

Then $a_{\overline{8}|} = 7.057233$ at that rate, so $X = 5000/7.057233 = €\,708.49$.

n	o/s at start	interest	capital	o/s at end
1	5000.00	143.69	564.80	4435.20
2	4435.20	127.46	581.03	3854.17

Note $143.69 = 0.0287373 \times 5000$, and *not* 0.03×5000: a quarter of 12% would be the nominal rate, not the effective one.

Solution 2

The symbol is defined exactly when nm is a whole number, so that the term holds a whole number of payment intervals:

	nm	whole?	defined?
$a^{(2)}_{\overline{23.25}\vert}$	46.5	no	no
$a^{(4)}_{\overline{23.25}\vert}$	93	yes	yes: 93 payments of $\frac{1}{4}$
$a^{(12)}_{\overline{7.25}\vert}$	87	yes	yes: 87 payments of $\frac{1}{12}$
$a^{(4)}_{\overline{7.4}\vert}$	29.6	no	no

In the two undefined cases the term ends part-way through a payment interval, and there is no agreed convention for what the last, incomplete instalment should be.

Solution 3

The effective quarterly rate is the one that compounds four times to 8%:

$$(1.08)^{1/4} - 1 \;=\; 0.019427, \qquad \text{so} \qquad i^{(4)} \;=\; 4 \times 0.019427 \;=\; 0.077706.$$

The interest due in one quarter on a balance of €20,000.00 is therefore

$$0.019427 \times 20{,}000 \;=\; €388.53 \qquad \left(= \; \frac{i^{(4)}}{4} \times 20{,}000 \right).$$

Using $i/4 = 2\%$ instead would have given €400.00. This would have been an overcharge of €11.47 a quarter.