Lecture 9: Mathematics of Finance

MA208 Quantitative Techniques for Business

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Unit III: Mathematics of Finance

In this unit we will cover

- Simple Interest,
- Compound Interest, Continuous Compound Interest,
- Future Value of an Annuity, Sinking Funds,
- Present Value of an Annuity, Amortization.

Lecture 10 - Outline

Today we will talk about

- Simple Interest,
- Compound Interest,
- Continuous Compound Interest.

Simple Interest

Simple interest is generally used only on short term loans or deposits.

If you deposit or borrow a sum of money P (called the principal), a fee (called interest) is charged. This is a fee charged to you for borrowing money from a bank or a fee the bank pays you for the use of money you have deposited in a savings account. It's usually computed as a percentage, called the interest rate, of the principal over a given period of time.

Simple interest is calculated using the following formula:

Simple Interest

$$I = Prt$$

where $I =$ interest, $P =$ principal, $r =$ simple interest rate (as a decimal) and $t =$ time in years.

Example

The interest on a loan of €500 at 12% per annum for nine months is

$$\begin{aligned} I &= Prt \\ &= (500)(0.12)(0.75) \\ &= €45. \end{aligned}$$

Note that nine months as a fraction of one year is $\frac{9}{12} = 0.75$.

At the end of nine months the borrower must repay the principal plus the interest, in total €545.

In general, if a principal P is borrowed at a rate r, then after t years the borrower will owe the lender an amount A that includes the principal P plus the interest I:

$$\begin{aligned} A &= P + I \\ &= P + Prt \\ &= P(1 + rt) \end{aligned}$$

Simple Interest Formula

$$A = P(1 + rt)$$

Note: We call A the future amount or future value, and the principal P is often called present value.

Example

Find the total amount due on a loan of €800 at 9% simple interest after 4 months.

Solution

$$P = 800€, \quad r = 9\% = 0.09, \quad t = \frac{4}{12} = \frac{1}{3}, \quad A = ?$$

$$\begin{aligned} A &= P(1 + rt) \\ A &= 800\left(1 + 0.09 \cdot \frac{1}{3}\right) \\ &= 800(1 + 0.03) \\ &= 824 \end{aligned}$$

Example

A family member has loaned you €1,000 with the understanding that the principal plus 4% p/a simple interest are to be repaid when you are able. How much would you owe if you repaid the loan after

(i) 2 years?

(ii) 10 years?

Solution

$$P = 1000€, \quad r = 0.04, A = P(1 + rt)$$

$t = 2$

$$\begin{aligned} A &= 1000(1 + 0.04 \cdot 2) \\ &= 1000 \cdot (1.08) \\ &= 1080€ \end{aligned}$$

Solution

t = 10

$$\begin{aligned} A &= 1000(1 + 0.04 \cdot 10) \\ &= 1000 \cdot 1.4 \\ &= 1400 \end{aligned}$$

Example

Your current car loan will be repaid in nine months time. At that time you intend to purchase a newer model and expect that with the trade in value of the current vehicle you will require an additional €5000.
Your local credit union just announced a new short term savings account with an interest rate of 10%.
How much should you deposit now in order to have a total of €5000 after nine months?

Solution

$$A = 5000€, \quad r = 0.1, \quad t = \frac{9}{12} = 0.75$$

$$\begin{aligned} A &= P(1+rt) \\ 5000 &= P(1+0.1\cdot 0.75) \\ &= P\cdot(1.075) \end{aligned}$$

Solution

This implies

$$P = \frac{5000}{1.075} = 4651.16$$

Compound Interest

If at the end of a payment period the interest due is reinvested, then the interest as well as the principal will earn interest during the next payment period. Interest paid on interest reinvested is called compound interest.

Let's look at an example to obtain a formula that allows us to carry out calculations involving compound interest.

Example

You deposit €1,000 in a bank that pays 10% p/a compounded quarterly. How much will your deposit be worth in 1 year?

Solution

Using the simple interest formula let's calculate the interest earned in each quarter.

In the first quarter $P = €1,000$, $r = 0.1$ and $t = 0.25$. So the amount at the end of the first quarter is

$$\begin{aligned} A &= P(1+rt) \\ &= 1,000(1+(0.1)(0.25)) \\ &= 1,000(1+0.025) = €1,025. \end{aligned}$$

At the beginning of the second quarter the principal is $P = €1,025$. And the amount at the end of the second quarter is

$$\begin{aligned} A &= P(1+rt) \\ &= 1,025(1+(0.1)(0.25)) \\ &= 1,025(1+0.025) = €1,050.63. \end{aligned}$$

At the beginning of the third quarter the principal is $P = €1,050.63$.
And the amount at the end of the third quarter is

$$\begin{aligned} A &= P(1+rt) \\ &= 1,050.63(1+(0.1)(0.25)) \\ &= 1,050.63(1+0.025) = €1,076.90. \end{aligned}$$

At the beginning of the fourth quarter the principal is $P = €1,076.90$.
And the amount at the end of the fourth quarter is

$$\begin{aligned} A &= P(1+rt) \\ &= 1,076.90(1+(0.1)(0.25)) \\ &= 1,076.90(1+0.025) = €1,103.82. \end{aligned}$$

If we trace the calculations backwards through the last example we can see that the amount at the end of the fourth quarter was

$$\begin{aligned}
€1,103.82 &= 1,076.90(1+0.025) \\
&= 1,050.63(1+0.025)(1+0.025) \\
&= 1,025(1+0.025)(1+0.025)(1+0.025) \\
&= 1,000(1+0.025)(1+0.025)(1+0.025)(1+0.025) \\
&= 1,000(1+0.025)^4,
\end{aligned}$$

where the number of compounding periods is 4, the principal is $€1,000$ and the interest rate per compounding period is $i = 0.025$.

In general, we have the following formula:

Compound Interest Formula

The amount A to which a principal P will grow after n compounding periods at a rate of i per compounding period is given by

$$A = P(1+i)^n.$$

Example

If €1000 is invested at 8% p.a. compounded

(i) annually,

(ii) quarterly,

(iii) monthly,

what is the amount after 5 years?

Solution

Compound interest formula: $A = P(1+i)^n$. Here,
$P = 1000€, \quad A = ?$.

1. Compound anually $\Rightarrow n = 5, i = 0.08$ per period.

$$A = 1000(1+0.08)^5 = 1469.33.$$

2. Compound quartally $\Rightarrow n = 4 \times 5 = 20, i = \frac{0.08}{4} = 0.02$.

$$A = 1000(1+0.02)^{20} = 1485.95.$$

3. Compound monthly $\Rightarrow n = 12 \times 5 = 60, i = \frac{0.08}{12}$ per period.

$$A = 1000\left(1+\frac{0.08}{12}\right)^{60} = 1489.95.$$

Continuous Compound Interest

What would happen to the amount if interest were compounded daily, or every minute, or every second?

Let's see what happens if the number of compounding periods per year, m, increases without bound.

Continuous Compound Interest

$$A = P(1+i)^n$$

Substitute $i = \frac{r}{m}, n = mt$

$$A = P(1+\frac{r}{m})^{mt}$$

Multiply the exponent by $\frac{r}{r}(=1)$

$$A = P(1+\frac{r}{m})^{\frac{m}{r}rt}$$

Let $x = \frac{m}{r}$, then $\frac{1}{x} = \frac{r}{m}$

$$A = P(1+\frac{1}{x})^{xrt}$$

Use a law of exponents: $a^{xy} = (a^x)^y$

$$A = P[(1+\frac{1}{x})^x]^{rt}$$

If $m \to \infty$, then $x \to \infty$ and

if $x \to \infty$, then $(1 + \frac{1}{x})^x \to e \approx 2.7183$.

So we get the following formula for continuous compound interest:

Continuous Compound Interest Formula

If a principal P is invested at an annual rate r (expressed as a decimal), then the amount A at the end of t years is given by

$$A = Pe^{rt}.$$

Continuous Compound Interest

Example

What amount will an account have after two years if €5000 is invested at an annual rate of 8%

(i) compounded daily?

(ii) compounded continuously?

Solution

i) Use compound interest formula

$$A = P(1+i)^n = P\left(1+\frac{r}{m}\right)^{mt}.$$

In our case, it is

$$P = 5000, \quad r = 0.08, \quad m = 365, \quad t = 2, \quad i = \frac{0.08}{365}, \quad n = 365 \times 2$$

$$\begin{aligned} A &= 5000\left(1+\frac{0.08}{365}\right)^{365\times 2} \\ &= 5867.45 \end{aligned}$$

ii) Using the continuous compound formula

$$A = Pe^{rt}$$

$$A = 5000e^{0.08\times 2} = 5867.55$$