Lecture 7: Probability

MA208 Quantitative Techniques for Business

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Lecture 7 - Outline

- Mutually exclusive (disjoint) events,
- De Morgan's Laws,
- Random variables,
- Probability distribution,
- Discrete / continuous random variables.

Example

Two balls are drawn successively without replacement from a box which contains 4 white balls and 3 red balls. Find the probability that

(a) the first ball drawn is white and the second is red,

(b) both balls are red.

Solution (a): first white, second red

Let $E_1 =$ “the first ball drawn is white” and $E_2 =$ “the second ball drawn is red”.

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The probability that the first ball is white is

$$P(E_1) = \frac{4}{7}.$$

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Since the ball is not replaced, 3 white + 3 red = 6 balls remain for the second pick.

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Since the ball is not replaced, 3 white + 3 red = 6 balls remain for the second pick. Thus, the probability that the second ball is red is

$$P(E_2 \mid E_1) = \frac{3}{6} = \frac{1}{2}.$$

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$$P(E_2 \mid E_1) = \frac{3}{6} = \frac{1}{2}.$$

Therefore, the required probability is

$$P(E_1 \cap E_2) = P(E_1)\,P(E_2 \mid E_1) = \frac{4}{7} \times \frac{1}{2} = \boxed{\frac{2}{7}}.$$

Solution (b): both red

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Since the ball is not replaced, 4 white + 2 red = 6 balls remain for the second pick.

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$$P(E_2 \mid E_1) = \frac{2}{6} = \frac{1}{3}.$$

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Examples

Suppose that the probability that a checked suitcase is lost on a single flight is

$$P_\ell = \frac{6}{10^3} = \frac{6}{1000}.$$

If a businesswoman takes 20 flights per year, what is the probability that at least one of her suitcases is lost within the next 20 years?

Assumptions:

- Different flights are independent;
- She checks a suitcase on every flight.

Solution

Let $\mathrm{E} =$ “a suitcase is lost on a given flight”.

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The probability that a suitcase is **not** lost on a single flight is

$$1 - P_\ell = 1 - \frac{6}{10^3} = \frac{994}{1000}.$$

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Therefore, the probability that at least one suitcase is mishandled is

$$1 - \left(\frac{994}{1000}\right)^{400}.$$

Remark

The key point is that the question asks for **at least one suitcase lost** in n flights. One might wonder: why not just take

$$P(E)^n = \left(\frac{6}{1000}\right)^{400} ?$$

The reason is subtle: $P(E)^n$ would correspond to the probability that **every single flight has a suitcase lost**. But we are not asking for "all lost," we are asking for "at least one lost."

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By contrast, if we define E = "a suitcase is lost on a flight", then

$$1 - P(E) = \text{probability that a suitcase is NOT lost on a flight.}$$

If we raise $1 - P(E)$ to the nth power, $(1 - P(E))^n$, we get the probability that **no suitcases are lost in any of the n flights**. This is exactly the opposite of "at least one lost." Therefore, to answer the original question, we subtract:

$$P(\text{at least one lost}) = 1 - (1 - P(E))^n.$$

In short:

Raising $P(E)$ to n does not give "at least one success", it gives the probability that **every trial is a success**.

Raising $1 - P(E)$ to n gives "no successes," and subtracting from 1 gives the desired "at least one."

Mutually Exclusive Events

Two or more events are said to be mutually exclusive (disjoint), if the occurence of any one of them means the others will not occur. That is, two or more events can not occur at the same time.

Example

Throwing a die, the events '4' and '5' are mutually exclusive.

Facts

- If E_1 and E_2 are mutually exclusive events, then
$$P(E_1 \cap E_2) = 0.$$
- If E_1 and E_1 are mutually exclusive events, then
$$P(E_1 \cup E_2) = P(E_1) + P(E_2).$$

Mutually Exclusive Events

Examples

1. Let E_1 be BIS students and E_2 be Arts students in this room. What is $P(E_1 \cap E_2)$ and $P(E_1 \cup E_2)$?

Solution

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1. No overlap, so $P(E_1 \cap E_2) = 0$ and $P(E_1 \cup E_2) = P(E_1) + P(E_2)$.

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2. Let E_3 be students in the swimming team and E_4 be students in the debating team. What is $P(E_3 \cap E_4)$ and $P(E_3 \cup E_4)$?

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   $P(E_3 \cap E_4)$ is what it is.

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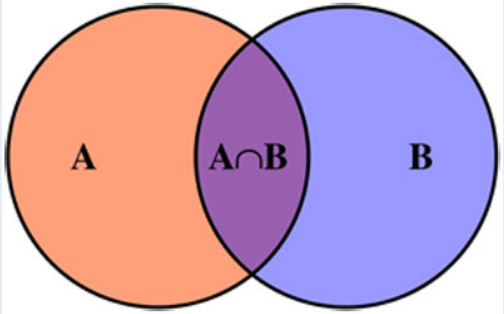
2. 

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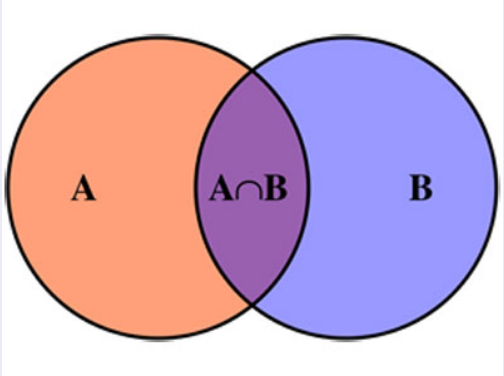
2. 

   $P(E_3 \cap E_4)$ is what it is.

   $P(E_3 \cup E_4) = P(E_3) + P(E_4) - P(E_3 \cap P(E_4))$

   We must not count $E_3 \cap P(E_4)$ twice!

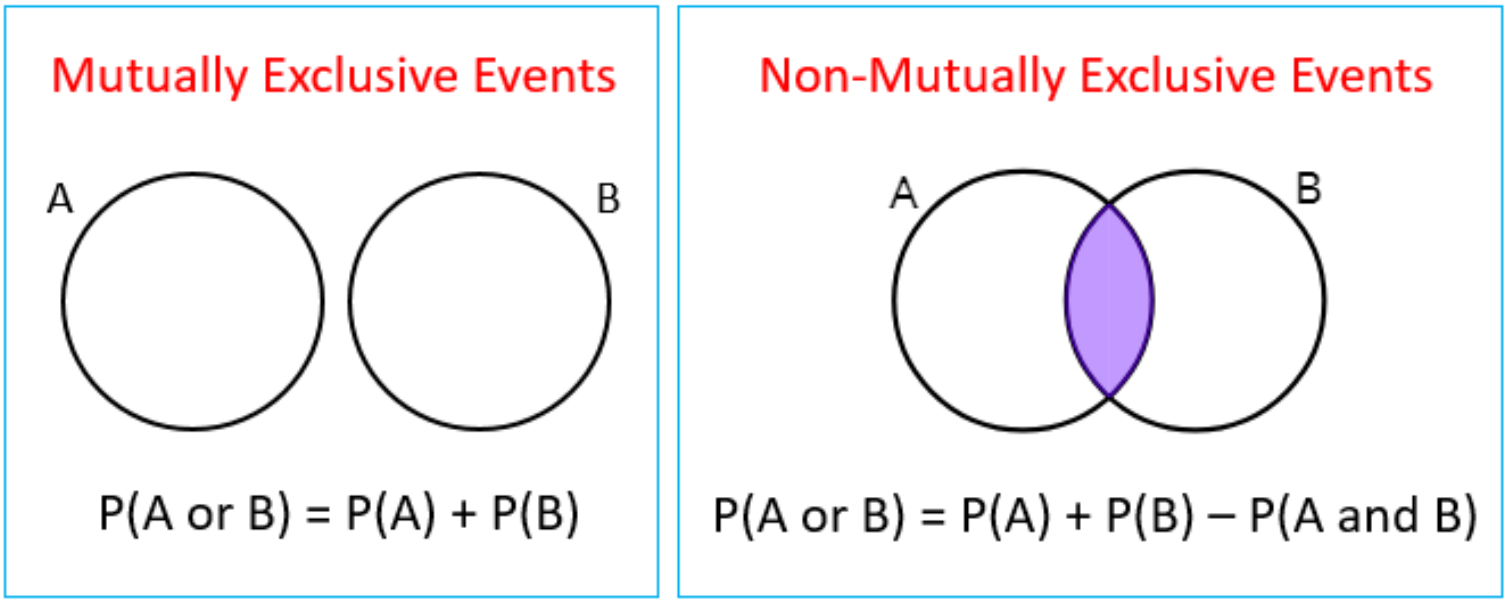
Mutually Exclusive Events

Facts

If If E_1 and E_2 are **not** mutually exclusive events, then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

This is best explained with a diagram:



Finding Probabilities

Example

Consider the set of numbers $\{1, 2, 3, ..., 10\}$ and the following events:

- Event A: I pick an even number,
- Event B: I pick a multiple of 3,
- Event C: I pick a multiple of 7.

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2. What about events A and C?
3. Calculate $P(A \cap B)$ and $P(A \cup B)$.
4. Calculate $P(A \cap C)$ and $P(A \cup C)$.

Solution

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \qquad |S| = 10$$

$$A = \{2, 4, 6, 8, 10\} = \text{"even number"} \qquad P(A) = \frac{5}{10} = \frac{1}{2}$$

$$B = \{3, 6, 9\} = \text{"multiple of 3"} \qquad P(B) = \frac{3}{10}$$

$$C = \{7\} \qquad P(C) = \frac{1}{10}$$

1. Clearly A, B are not disjoint as $6 \in A$ and $6 \in B$.

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$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{10} + \frac{3}{10} - \frac{1}{10} = \frac{7}{10}$

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$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{10} + \frac{3}{10} - \frac{1}{10} = \frac{7}{10}$
4. $P(A \cap C) = 0$
$P(A \cup C) = P(A) + P(C) = \frac{5}{10} + \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$

De Morgan's Laws

Rules from set theory can be helpful to calculate probabilities, for example:

De Morgan's Laws

Let A and B be sets. Then

$$\overline{A \cup B} = \overline{A} \cap \overline{B},$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

Example

Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with subsets $A = \{1, 2, 4, 6, 8, 10\}$ and $B = \{2, 3, 5, 10\}$. Verify De Morgan's Laws for A and B.

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$$\overline{B} = \{1, 4, 6, 7, 8, 9\}$$

$$\overline{A} \cap \overline{B} = \{7, 9\}$$

$$\overline{A} \cup \overline{B} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$$

$$A \cap B = \{2, 10\}$$

$$\overline{A \cup B} = \{7, 9\}$$

$$\overline{A \cap B} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today or tomorrow.

Finding Probabilities

Suppose we have the following information:

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2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today or tomorrow.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

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Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Finding Probabilities

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Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ We don't know $P(A \cap B)$!

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today or tomorrow.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

We have to use another method... Let's try De Morgan's Laws.

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

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Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$P(A \cup B) = 1 - P(\overline{A \cup B})$ $\qquad$ $P(X) + P(\overline{X}) = 1$ Always!

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today or tomorrow.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$$P(A \cup B) = 1 - P(\overline{A \cup B}) \qquad P(X) + P(\overline{X}) = 1 \quad \text{Always!}$$
$$= 1 - P(\overline{A} \cap \overline{B}) \qquad \text{De Morgan}$$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today or tomorrow.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$$\begin{aligned} P(A \cup B) &= 1 - P(\overline{A \cup B}) && P(X) + P(\overline{X}) = 1 \quad \text{Always!} \\ &= 1 - P(\overline{A} \cap \overline{B}) && \text{De Morgan} \\ &= 1 - 0.3 = 0.7 \end{aligned}$$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today and tomorrow.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
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Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
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3. There is a 30% chance that it does not rain either day.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.6 + 0.5 - 0.7 \\ &= 0.4 \end{aligned}$$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
3. There is a 30% chance that it does not rain either day.

What is the probability that it will rain today but not tomorrow.

Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

Finding Probabilities

Suppose we have the following information:

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Solution

Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$P(A \cap \overline{B}) = P(A - B)$

Finding Probabilities

Suppose we have the following information:

1. There is a 60% chance that it will rain today.
2. There is a 50% chance that it will rain tomorrow.
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$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$$\begin{aligned} P(A \cap \overline{B}) &= P(A - B) \\ &= A(A) - P(A \cap B) \end{aligned}$$

Draw a Venn-diagram to check

Finding Probabilities

Suppose we have the following information:

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Let A = "rain today", B = "rain tomorrow".

$$P(A) = 0.6, \qquad P(B) = 0.5, \qquad \text{and} \qquad P(\overline{A} \cap \overline{B}) = 0.3.$$

$$\begin{aligned} P(A \cap \overline{B}) &= P(A - B) \\ &= A(A) - P(A \cap B) \\ &= 0.6 - 0.4 = 0.2 \end{aligned}$$

Draw a Venn-diagram to check

Random Variables

A **random variable** is a numerical variable whose value is determined by an underlying random experiment.

Example

I toss a coin five times. This is a random experiment and the sample space can be written as

$$S = \{\text{TTTTT}, \text{TTTTH}, ..., \text{HHHHH}\}.$$

Note that here the sample space has $2^5 = 32$ elements. Suppose that in this experiment, we are interested in the number of heads. We can define a random variable X whose value is the number of observed heads. The value of X will be one of $0, 1, 2, 3, 4$ or 5 depending on the outcome of the random experiment.

Probability Distribution

A random variable has a **probability distribution**, which specifies the probability that its value falls in any given interval.

Example

Random variable	Probability
10 - 19	0.04
20 - 29	0.08
30 - 39	0.14
40 - 49	0.20
50 - 59	0.32
60 - 69	0.16
70 - 79	0.06

The random variable is the exam result received by a student.

Discrete / Continuous Random Variables

Depending on the type of the experiment, the numerical value that the random variable takes can be classified as **discrete random variable** or **continuous random variable**.

- **Discrete random variable**: the set of assumed values is *countable*, e.g. numbers of players in a team, number of visits in a doctor's surgery;
- **Continuous random variable**: the set of assumed values is *uncountable*, e.g. amount of rainfall in a month, the time required to cycle to university.

Discrete / Continuous Random Variables

Examples

(1) Two balls are drawn in random in a succession without replacement from an urn containing four red balls and six black balls. Find the probabilities of all possible outcomes.

Solution

Discrete / Continuous Random Variables

Solution

Step 1: List all possible outcomes Two balls are drawn one after the other. We use:

$$\mathrm{R} = \text{Red}, \quad \mathrm{B} = \text{Black}$$

All possible outcomes	Probability
RR	
RB	
BR	
BB	

Discrete / Continuous Random Variables

Solution

Step 2: Compute probabilities (4 + 6 = 10 balls in total)

All possible outcomes	Probability
RR	$\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$
RB	
BR	
BB	

Discrete / Continuous Random Variables

Solution

Step 2: Compute probabilities (4 + 6 = 10 balls in total)

All possible outcomes	Probability
RR	$\frac{2}{15}$
RB	$\frac{4}{10} \times \frac{6}{9} = \frac{4}{15}$
BR	
BB	

Discrete / Continuous Random Variables

Solution

Step 2: Compute probabilities (4 + 6 = 10 balls in total)

All possible outcomes	Probability
RR	$\frac{2}{15}$
RB	$\frac{4}{15}$
BR	$\frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$
BB	

Discrete / Continuous Random Variables

Solution

Step 2: Compute probabilities (4 + 6 = 10 balls in total)

All possible outcomes	Probability
RR	$\frac{2}{15}$
RB	$\frac{4}{15}$
BR	$\frac{4}{15}$
BB	$\frac{6}{10} \times \frac{5}{9} = \frac{1}{3}$

Discrete / Continuous Random Variables

Solution

Final check:

$$\frac{2}{15} + \frac{4}{15} + \frac{4}{15} + \frac{1}{3} = 1$$

So all probabilities are correct.

Discrete / Continuous Random Variables

Examples

(2) A jar coffee is picked at random from a filling process in which an automatic machine is filling coffee jars each with 1 kg of coffee.

Due to some faults in the automatic process, the weight of a jar could vary from jar to jar between 0.9 kg and 1.05 kg.

Let X denote the weight of a jar of coffee selected. What is the range of X?

Solution

This is a continuous random variable.

The range is $0.9 \leqslant x \leqslant 1.05$.

Probability Distributions

There are some specific distributions that are used over and over in practice, thus they have been given special names. There is a random experiment behind each of these distributions. Since these random experiments model a lot of real life phenomenon, these special distributions are used frequently in different applications. That's why they have been given a name and we devote some time to study two of them.

Do not get intimidated by the complicated looking formulas, look at each distribution as a practice problem on discrete random variables.

Binomial Probability Distribution

Example

Suppose that I have a coin with $P(H) = p = 0.5$. I toss the coin $n = 30$ times and define X to be the total number of heads that I observe. Then X is binomial with parameters n and p. The range of X in this case is $\{0, 1, 2, ..., 30\}$. What is the probability to get 17 heads?

We can use the formula for Binomial Probability Distribution (see next slide):

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

Example

$$P(X = 17) = \binom{30}{17} \cdot (0.5)^{17} \cdot (1-0.5)^{(30-17)} \approx 0.111535052$$

Binomial Probability Distribution

The probability distribution of the random variable is called a **binomial distribution** if

- the experiments consist of a fixed number of n trials,
- the probability of success (denoted by p) remains constant,
- all trials are independent,
- each trial is either a success or failure (hence the name binomial).

The binomial distribution can be described by

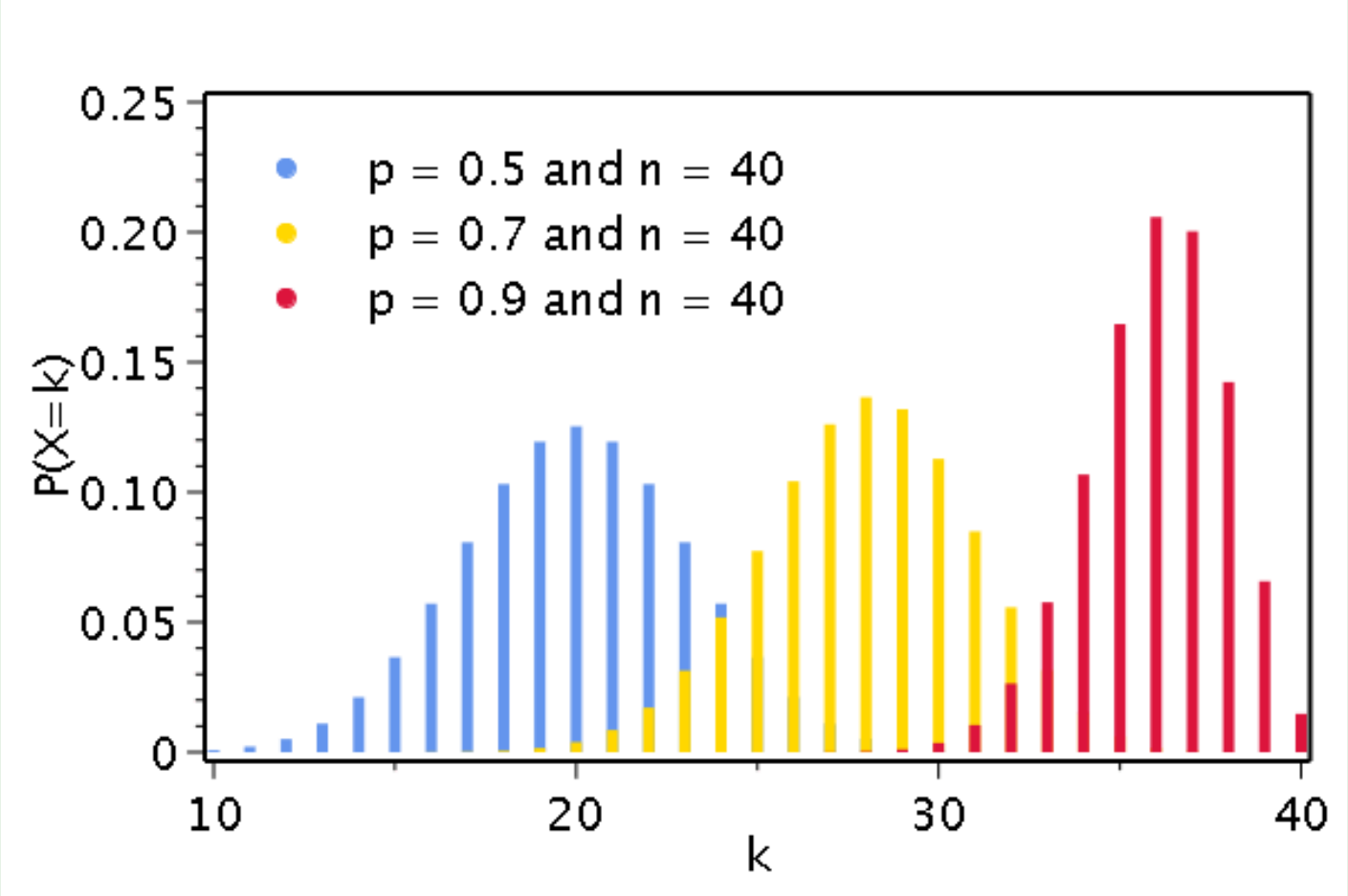
Binomial Probability Distribution

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where n is the number of trials, $x \in \{0, 1, 2, 3, ..., n\}$ is the discrete variable and p is the probability of success in a single trial.

Binomial Probability Distribution

Example



Binomial Probability Distribution

Example

Let 0.3 be the probability that a person will buy ice cream in a shop. Find the probability that among six people four will buy ice cream.

Binomial Probability Distribution

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Independent events
Success / failure
$\Longrightarrow$ Binomial distribution

Binomial Probability Distribution

Example

Let 0.3 be the probability that a person will buy ice cream in a shop. Find the probability that among six people four will buy ice cream.

Solution

$$\left.\begin{array}{l}\text{Independent events}\\ \text{Success / failure}\end{array}\right\} \Longrightarrow \text{Binomial distribution}$$

Let $X :=$ number of people who buy ice cream, $n = 6$ trials.

We want 4 successes, with $p = 0.3$.

Binomial Probability Distribution

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Let $X :=$ number of people who buy ice cream, $n = 6$ trials.

We want 4 successes, with $p = 0.3$. This gives us

$$P(X = 4) = \binom{6}{4}(0.3)^4(1-0.3)^{(6-4)}$$

Binomial Probability Distribution

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We want 4 successes, with $p = 0.3$. This gives us

$$P(X = 4) = \binom{6}{4}(0.3)^4(1-0.3)^{(6-4)}$$

$$= \binom{6}{4}(0.3)^4(0.7)^2 = 0.06$$

Binomial Probability Distribution

Example

Let 0.3 be the probability that a person will buy ice cream in a shop. Find the probability that among six people four will buy ice cream.

Solution

$$\left.\begin{array}{l}\text{Independent events}\\ \text{Success / failure}\end{array}\right\} \Longrightarrow \text{Binomial distribution}$$

Let $X :=$ number of people who buy ice cream, $n = 6$ trials.

We want 4 successes, with $p = 0.3$. This gives us

$$\begin{aligned} P(X = 4) &= \binom{6}{4}(0.3)^4(1-0.3)^{(6-4)} \\ &= \binom{6}{4}(0.3)^4(0.7)^2 = 0.06 \\ &= \frac{6!}{4! \cdot (6-4)!}(0.3)^4(0.7)^2 = 0.06 \end{aligned}$$

Binomial Probability Distribution

Example

Let 0.3 be the probability that a person will buy ice cream in a shop. Find the probability that among six people four will buy ice cream.

Solution

$$\left.\begin{array}{l}\text{Independent events}\\ \text{Success / failure}\end{array}\right\} \Longrightarrow \text{Binomial distribution}$$

Let $X :=$ number of people who buy ice cream, $n = 6$ trials.

We want 4 successes, with $p = 0.3$. This gives us

$$\begin{aligned} P(X = 4) &= \binom{6}{4}(0.3)^4(1-0.3)^{(6-4)} \\ &= \binom{6}{4}(0.3)^4(0.7)^2 = 0.06 \\ &= \frac{2\cdot 3\cdot 4\cdot 5\cdot 6}{2\cdot 3\cdot 4\cdot 2}(0.3)^4(0.7)^2 = 0.06 \end{aligned}$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 0$.

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 0$.

$$P(x = 0) = \binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3$$

Binomial Probability Distribution

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A die is tossed three times. What is the probability of

(i) no fives?

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Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 0$.

$$P(x = 0) = \binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3$$

$$= \frac{3!}{0! \cdot (3-0)!} \cdot 1 \cdot \left(\frac{5}{6}\right)^3$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

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Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 0$.

$$P(x = 0) = \binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3$$

$$= \frac{1 \cdot 2 \cdot 3}{1 \cdot 1 \cdot 2 \cdot 3} \left(\frac{5^3}{6^3}\right) = \frac{125}{216} \simeq 0.5758$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 1$.

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 1$.

$$P(x = 1) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 1$.

$$P(x = 1) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2$$

$$= \frac{3!}{1! \cdot (3-1)!} \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 1$.

$$P(x = 1) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2$$

$$= \frac{1 \cdot 2 \cdot 3}{1 \cdot 1 \cdot 2} \frac{1}{6} \left(\frac{5^2}{6^2}\right) = \frac{75}{216} \simeq 0.34722$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
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Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

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Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 3$.

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 3$.

$$P(x = 3) = \binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 3$.

$$P(x = 3) = \binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0$$

$$= \frac{3!}{3! \cdot (3-3)!} \cdot \left(\frac{1}{6}\right)^3 \cdot 1$$

Binomial Probability Distribution

Example

A die is tossed three times. What is the probability of

(i) no fives?

(ii) one five?

(iii) three fives?

Solution

Binomial distribution (5 or not 5), with $n = 3$, $p = \frac{1}{6}$.
Let $X =$ "number of 5 appearing.

(i) $x = 3$.

$$P(x = 3) = \binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0$$

$$= \cdot \frac{1}{6^3} = \frac{1}{216} \simeq 4.6296 \times 10^{-1}$$

Binomial Probability Distribution

Example

A blindfolded marksman finds that on average he hits the target four times out of five. If he fires four shots, what is the probability of

(i) more than two hits?

(ii) at least three misses?

Binomial Probability Distribution

Example

A blindfolded marksman finds that on average he hits the target four times out of five. If he fires four shots, what is the probability of

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Solution

Hit / miss $\Longrightarrow$ 2 possible outcomes $\Longrightarrow$ Binomial Distribution.

Binomial Probability Distribution

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Solution

Hit / miss $\Longrightarrow$ 2 possible outcomes $\Longrightarrow$ Binomial Distribution.

$n =$ $p =$ $X :=$

Binomial Probability Distribution

Example

A blindfolded marksman finds that on average he hits the target four times out of five. If he fires four shots, what is the probability of

(i) more than two hits?

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Solution

Hit / miss $\Longrightarrow$ 2 possible outcomes $\Longrightarrow$ Binomial Distribution.

$$n = 4 \qquad p = \frac{4}{5} = 0.8 \qquad X := \text{number of hits}$$

(i) $P(x \geqslant 2) = P(x = 3) + P(x = 4)$

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