Lecture 17: Linear Programming - The Corner Point Method

MA208 Quantitative Techniques for Business

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Recall from last lecture:

Linear programming is a mathematical optimisation technique, where optimisation means maximizing or minimizing a linear equation subject to some constraints.

We learned how to formalise a linear programming problem. Today we will learn a method to solve this type of problems, the **Corner Point Method**.

Let's look at one of the examples from last week.

Example

A factory makes two types of machine, A and B. The table below details resource requirements for one of each type of machine.

Resource	Machine A	Machine B	Monthly availability
Energy	100 kWh	200 kWh	20,000 kWh
Steel	60 kg	80 kg	10,000 kg
Labour	2.5 h	2 h	400 h

The company makes a profit of €48 per machine A and €38 per machine B produced.

- Formulate this problem as a linear programming problem.
- How many of each type of machine should the company produce to maximize profit?

Solution

Let x be the number of machine A and y be the number of machine B.

The total profit can be described by the objective function:

$$p = 48x + 38y.$$

We must maximise this function subject to the following constraints:

$$\begin{array}{rcrcr} 100x & + & 200y & \leqslant & 20,000 \\ 60x & + & 80y & \leqslant & 10,000 \\ 2.5x & + & 2y & \leqslant & 400 \end{array}$$

We also assume that $x \geqslant 0$ and $y \geqslant 0$, where x and y are integers.

The Corner Point Method

Corner Point Method

To solve a linear programming problem with two decision variables:

1. Graph the region satisfying all of the inequalities. This is called region of feasible solutions.
2. Find the co-ordinates of all of the corner points of the region.
3. Substitute each into the function to be maximised.
4. One of these is the answer!

Note: To solve linear programming problems with more than two variables, more advanced methods are required, for example the "Simplex Method".

Solution ctd.

Recall: we must

maximize

$$p = 48x + 38y,$$

subject to

$$\begin{array}{lrcrcr} (1) & 100x & + & 200y & \leqslant & 20,000; \\ (2) & 60x & + & 80y & \leqslant & 10,000; \\ (3) & 2.5x & + & 2y & \leqslant & 400. \end{array}$$

First, find the feasible region. To do that, draw graphs of the three inequalities.

The inequalities $x \geqslant 0$ and $y \geqslant 0$ indicate that the solution will lie in the first quadrant.


200
150
100
50
0
y
0
50
100
150
200
x

Now find the coordinates of the four corner points.

Fact: The optimal solution must occur at one of the corner points.

We know already two of the corner points, namely those on the x- and y axis:

$$(0, 100)\ ,\ (160, 0).$$

The other two corner points are the intersections of lines (1) and (2) and the intersection of lines (2) and (3).

Thus, we have to find those.

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Line intercepts (1) and (3):

$$100 - 0.5x = 200 - 1.25x \Rightarrow x = 133.33,\ y = 33.33$$

It violates (2) Line intercepts of (2) and (3):

$$125 - 0.75x = 200 - 1.25x \Rightarrow x = 150,\ y = 12.5$$

The intersection point of lines (1) and (2) is $(100, 50)$, and the intersection point of lines (2) and (3) is $(150, 12.5)$.

So the **corner points** are

$$(0, 100), (160, 0), (100, 50), (150, 12.5).$$

To find out which of those is the solution, we need to substitute each of them into the profit function

$$p = 48x + 38y.$$

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- $(0, 100)$ $\quad p = 48(0) + 38(100) = 3,800$
- $(160, 0)$ $\quad p = 48(160) + 38(0) = 7,680$
- $(100, 50)$ $\quad p = 48(100) + 38(50) = 6,700$
- $(150, 12.5)$ $\quad p = 48(150) + 38(12.5) = 7,675$

So the maximum profit is $€7,680$ which happens when the factory makes 160 machines of type A and 0 machines of type B.

Example (from 2015 exam paper)

An oil company wish to inject a fracking fluid into the ground of an island and can use two types of chemicals, Bromine and DBNPA. Both types have an impact on the environment and therefore there are limits on the impact type.

Impact type	**Bromine**	**DBNPA**	**Limit for permitted damage**
Drinking water	40 kTpU	20 kTpU	1200 kilotons
Air pollution	30 kTpU	30 kTpU	1800 kilotons
Surface contamination	25 kTpU	50 kTpU	1500 kilotons

(kTpU = kilotons per unit)

Example (ctd.)

(a) Let x and y denote the number of units to be extracted with Bromine (resp. DBNPA). Write down three inequalities representing the constraints on x and y imposed by the above table.

(b) Sketch these inequalities and shade the region of feasible solutions.

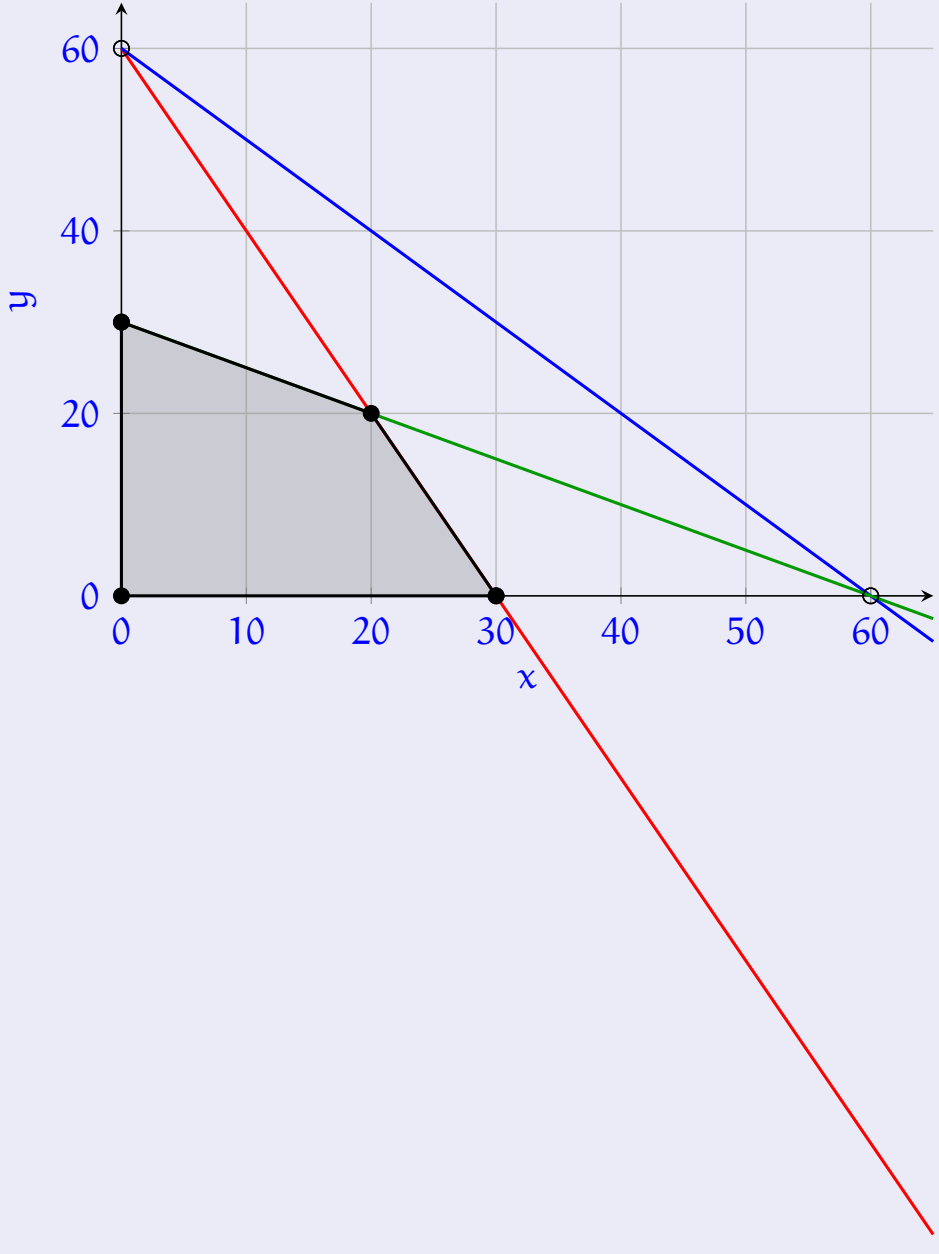
(c) The company makes a profit of €13 million per unit of gas extracted with Bromine and of €7 million per unit extracted with DBNPA.
How many units extracted with Bromine together with how many extracted with DBNPA do maximize the profits?
And what is the maximum profit?

Solution

a)

$$
\begin{aligned}
40x + 20y &\leqslant 1200 && (1)\\
30x + 30y &\leqslant 1800 && (2)\\
25x + 50y &\leqslant 1500 && (3)\\
x &\geqslant 0 && (4)\\
y &\geqslant 0 && (5)
\end{aligned}
$$

b)



Corner points are (0,30), (30, 0) and the intersections of lines 1 and 3. We have

$$60 - 2x = 30 - 0.5x \Rightarrow x = 20, ; y = 20$$

Profits:

$$\begin{aligned} (30, 0) &: \quad p = 13 \cdot 30 + 7 \cdot 0 = 390 \\ (0, 30) &: \quad p = 13 \cdot 0 + 7 \cdot 30 = 210 \\ (20, 20) &: \quad p = 13 \cdot 20 + 7 \cdot 20 = 400 \end{aligned}$$

Example

The annual subscription for a tennis club is €20 for adults and €8 for juniors. The club needs to raise at least €800 in subscriptions to cover its expenses.
The total number of members is restricted to 50. The number of junior members is to be between one quarter and one third of the number of adult members.

(i) Represent this problem graphically.

(ii) Find the number of adults and junior members which will bring the largest amount of money in subscriptions.

Solution