Lecture 12: Examples

MA208 Quantitative Techniques for Business

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Recall formulae from our last three lectures:

- **Simple Interest:**

$$A = P(1 + rt) \quad \text{and} \quad I = Prt,$$

- **Compound Interest:**

$$A = P(1 + i)^n,$$

- **Continuous Compound Interest:**

$$A = Pe^{rt},$$

where $I =$ interest, $A =$ amount, $P =$ principal,
$r =$ annual interest rate, $t =$ time in years,
$i =$ interest rate per compounding period,
$n =$ number of compounding periods.

- **Present Value of an Ordinary Annuity:**

$$PV = PMT\frac{1-(1+i)^{-n}}{i},$$

- **Future Value of an Ordinary Annuity:**

$$FV = PMT\frac{(1+i)^{n}-1}{i},$$

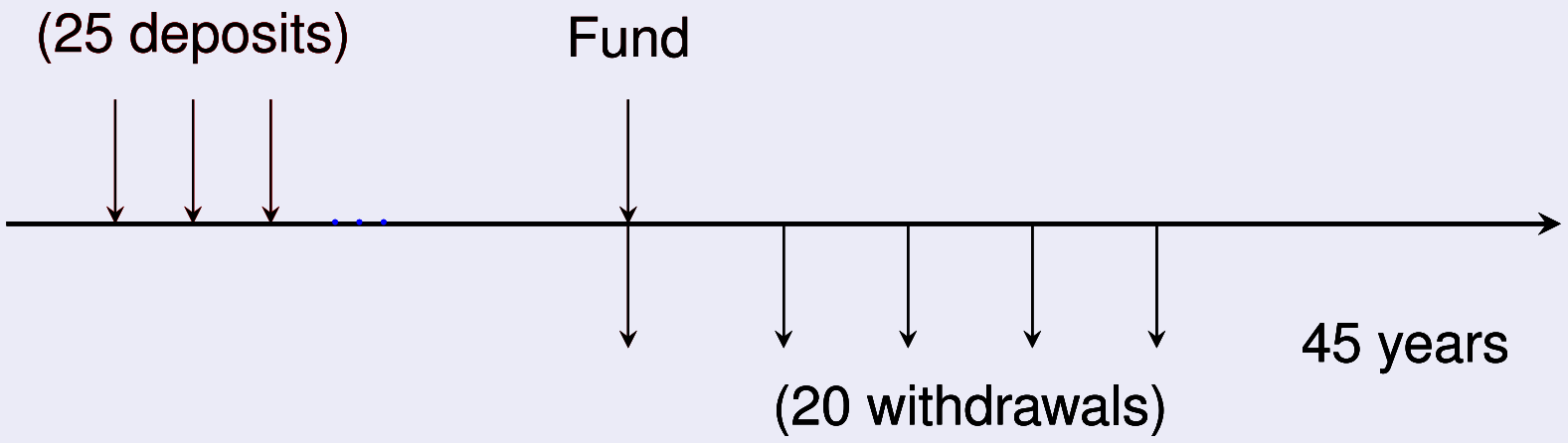
where
$PV =$ present value, $FV =$ future value, $PMT =$ periodic payment,
$i =$ interest rate per compounding period,
$n =$ number of payments, *with the understanding that payments are made at the end of each period.*

Example

You wish to purchase a retirement plan that earns 6.5% interest compounded annually. You will make annual deposits into this account for 25 years, and then make 20 equal withdrawals of $€10,000$ reducing the balance to zero. Compute

1. the value of the fund based on the withdrawals and
2. the amount of each deposit in order to maintain the fund,
3. the total interest earned over the entire 45 years.

Solution



(1) Use the 'Present Value' Formula to calculate the fund value:

$$\begin{aligned} PV &= PMT\frac{1-(1+i)^{-n}}{i} \\ &= 10000\frac{1-(1+0.065)^{-20}}{0.065} \\ &= €\,110,185.07 \end{aligned}$$

(2) Now use the 'Future Value' Formula to calculate the amount of the deposits:

$$FV = PMT\frac{(1+i)^{n}-1}{i}$$

We have FV but need PMT

$$\begin{aligned} \text{PMT} &= \text{FV}\frac{i}{(1+i)^n - 1} \\ &= 110185.07\frac{0.065}{(1.065)^{25} - 1} \\ &= €\,1,871.06 \qquad \text{is the yearly deposit.} \end{aligned}$$

$$\begin{aligned} \text{(3) Total interest} &= \text{(total withdrawals) - (total deposits)} \\ &= (20 \times 10000) - (25 \times 1871.06) \\ &= 200000 - 46776.50 \\ &= €\,153,223.50 \end{aligned}$$

is the total interest earned over 45 years.

Revision

Example

How long, to the nearest day, will it take $€\,20,000$ to grow to $€\,300,000$ at 9% annual interest compounded monthly?

Solution

Use compound interest formula:

$$A = P(1+i)^n.$$

We have

$$A = 300,000, \quad P = 20,000, \quad i = \frac{0.09}{12} = 0.0075, \quad n = 12x.$$

Find x.

$$
\begin{aligned}
300,000 &= 20,000\left(1+\frac{0.09}{12}\right)^{12x} \\
\frac{300,000}{20,000} &= \left(1+\frac{0.09}{12}\right)^{12x} \\
15 &= 1.0075^{12x} \\
\ln(15) &= \ln(1.0075^{12x}) \\
\ln(15) &= 12x\ln(1.0075) \\
12x &= \frac{\ln(15)}{\ln(1.0075)} \\
x &= \frac{1}{12}\frac{\ln(15)}{\ln(1.0075)} \approx 30.202
\end{aligned}
$$

Answer: 30 years and $365 \times 0.2 = \frac{365}{5} \approx 73$ days.

Example

The first credit card that you get charges 10.29% interest to its customers and compounds that interest monthly. Within one day of getting your first credit card, you max out the credit limit by spending $€\,1,300.00$. If you do not buy anything else on the card and you do not make any payments, how much money would you owe the company after 6 months?

Solution

Compound interest formula: $A = P(1+i)^n$.
Here, $A = ?, \quad P = 1300, \quad i = \frac{0.10129}{12} = 0.008575, \quad n = 6.$

$$A = 1300(1 + 0.008575)^6 = 1368.34$$

College Fund With Gap Years

Example

A parent deposits €x at the end of each year for 10 years into an account earning 6% annually. The child waits 2 additional years before starting college.

The parent then withdraws €15,000 per year for 4 years.

What must x be?

Solution

Step 1: Required fund at start of college (12 Years after deposit of x.)

Use Present Value of an Ordinary Annuity:

$$\begin{aligned} PV_{12} &= €\,15000 \cdot \frac{1-(1.06)^{-4}}{0.06} \\ &= €\,15000 \cdot 3.465 \\ &= €\,51,975 \end{aligned}$$

Step 2: Determine the accumulated value of the deposits.

We now know that in year 12, there needs to be €51,975 on the bank account.

We now want to understand how much there needs to be at year 10, for the interest in 2 years to reach €51,975.

Compound interest

This is an "inverse" compound interest problem and we have

$$€\,51,975 = P_{10}(1.06)^2$$

which means that

$$P_{10} = \frac{€\,51,975}{1.1236} \approx €\,46,257.56$$

needs to accrued in the first 10 years.

Step 3: Determine the monthly deposits

Future Value of an Annuity

We use

$$FV = PMT\frac{(1+i)^n - 1}{i}$$

with $FV = FV_{10} = P_{10} = €\,46,257.56$, $i = 0.06$ and $n = 10$. We have

$$€\,46,257.56 = PMT \cdot \frac{(1.06)^{10} - 1}{0.06}$$

implying

$$€\,x = PMT = \frac{€\,46,257.56}{13.18} \approx €\,3509.67.$$

Car Replacement Fund

Example

You deposit €3,000 today into an account earning 5% annually.
Additionally, you deposit € x at the end of each year for 5 years.
At the end of Year 5, you want exactly €25,000.
What must x be?

Compounded Initial Deposit

Note that the initial deposit and the monthly deposits are independent. Hence, we first calculate how much money is accrued by the initial deposit, to see how much would be missing without deposits.

This is a compound interest deposit question:

$$A = 3000 \cdot (1 + 0.05)^5 = € 3828.84.$$

Hence, we still need

$$€ 25000 - € 3828.84 \approx € 21,171.15$$

Solution

Future Value of Annuity

We now have to determine how much we have to deposit every month to reach our target. This is a *future value of annuity* question.

$$€\,21,171.15 = \text{PMT} \cdot \frac{(1.05)^5 - 1}{0.05}$$

implying

$$\text{PMT} = \frac{€\,21,171.15 \times 0.05}{(1.05)^5 - 1} = 3831.44$$