Semester II 2026

MA135/MA160 - Algebra II



OLLSCOIL NA GAILLIMHE
UNIVERSITY OF GALWAY

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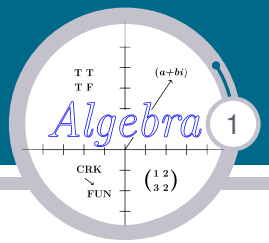
With special thanks to Prof. Aisling McCluskey & Dr. Róisín Hill

Lecture 1: Introduction to propositional logic

Course information



Module: MA135 or MA160

Course: Algebra II

Who am I?: Ilia Pirashvili, PhD

Lectures: Mon 1pm in AC202 and Tues 10am in McMunn

Slides: Slides will be uploaded shortly *before* each lecture (after this).

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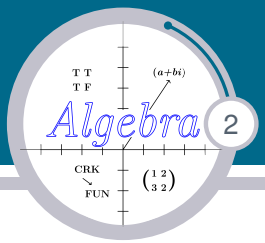
Office: Room 3001, Áras de Brún

Office hours: Tue 11am and Fr 14pm
(Or just drop by my office. Usually, I'll have time.)

Tutorials: There will be tutorials, but details will be communicated over the new few days.

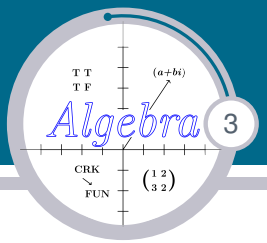
Course breakdown



- Propositional logic
- Complex numbers
- Systems of linear equations

Study advice: Please ensure that you study the relevant lecture material in advance of each lecture. This will help you engage better and be better prepared for your Tuesday in-class assignments.

Propositional logic

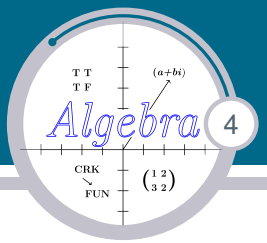


Consider the following text:

> *If school is cancelled, I will go into town with my friends.*
> *School is not cancelled.*
> *Therefore I will not go into town with my friends.*

This is an example of . . . what do you think?!

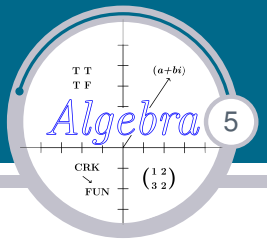
Propositional logic



The most basic sentences in propositional logic are *simple sentences*, i.e., sentences that are either TRUE (T) or FALSE (F). For example:

University of Galway is in Limerick.	F
A square has 4 sides.	T
Some people like listening to music.	T
Broccoli is a fruit.	F

Propositional logic and modular arithmetic



Last semester you studied modular arithmetic, e.g.,

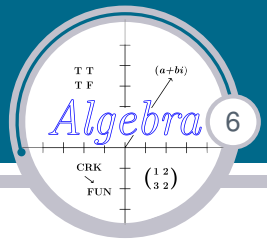
$$3 + 5 \equiv 1 \bmod 7$$
$$7 + 8 \equiv 3 \bmod 12$$
$$7 - 8 \equiv 11 \bmod 12$$

Now we are specifically interested in mod 2 arithmetic, e.g.,

$$3 + 5 \equiv 0 \bmod 2,$$
$$20 + 15 \equiv 1 \bmod 2,$$
$$233 \equiv 1 \bmod 2,$$
$$100 \equiv 0 \bmod 2.$$

There are only two possible outcomes, similar to propositional logic, i.e., 1 (T) or 0 (F).

Connective ¬



The *connective* negation is denoted as ¬.

Simple sentences:

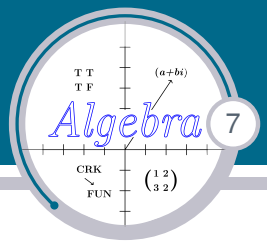
P_1	Galway is in Connacht.	T
P_2	Galway is in Munster.	F
Q_1	Broccoli is a fruit.	F
Q_2	Broccoli is a vegetable.	T

Their negations are:

$\neg P_1$	means Galway is not in Connacht.	F
$\neg P_2$	means Galway is not in Munster.	T
$\neg Q_1$	means broccoli is not a fruit.	T
$\neg Q_2$	means broccoli is not vegetable.	F

If P is true T then ¬P is false F. The negation connective ¬ is what is called a *unary operator* i.e. it operates on one sentence only.

Connective and



The connective and is denoted as $\wedge$.

Simple sentences:

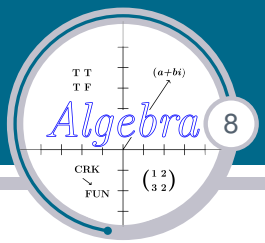
P_1	Galway is in Connacht.	T
P_2	Galway is in Munster.	F
Q_1	Broccoli is a fruit.	F
Q_2	Broccoli is a vegetable.	T

We use *connectives*, such as and, to combine simple sentences. E.g.,

$P_1 \wedge P_2$	Galway is in Connacht and Galway is in Munster.	F
$P_1 \wedge Q_1$	Galway is in Connacht and broccoli is a fruit.	F
$P_2 \wedge Q_1$	Galway is in Munster and broccoli is a fruit.	F
$P_1 \wedge Q_2$	Galway is in Connacht and broccoli is a vegetable.	T

Note that both of the sentences need to be true (T). The connective $\wedge$ is a binary operator i.e. it operates on two sentences.

Definition of and ($\wedge$)

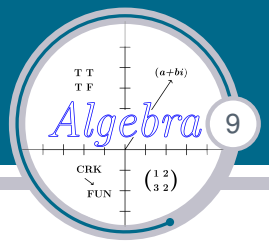


The meaning of and ($\wedge$) is captured by the following table, where P, Q are given sentences:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

We take this table as the **definition of** $\wedge$; the table is called a **truth table**.

Connective or



The connective or is denoted as $\vee$ and is treated as inclusive.

Simple sentences:

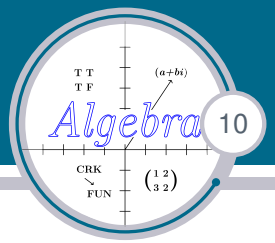
P_1	Galway is in Connacht.	T
P_2	Galway is in Munster.	F
Q_1	Broccoli is a fruit.	F
Q_2	Broccoli is a vegetable.	T

We may combine the above simple sentences using the connective or as follows:

$P_1 \vee P_2$	Galway is in Connacht or Galway is in Munster.	T
$Q_1 \vee Q_2$	Broccoli is a fruit or broccoli is a vegetable.	T
$P_1 \vee Q_2$	Galway is in Connacht or broccoli is a vegetable.	T
$P_2 \vee Q_1$	Galway is in Munster or broccoli is a fruit.	F

True T if one or both of the (simple) sentences are true T.

Definition of or ($\vee$)

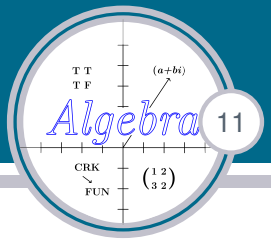


The connective or ($\vee$) is defined by the following truth table for sentences P, Q:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

In a similar way, we can easily create the truth table that defines the negation connective $\neg$:

Truth tables



Given sentences P and Q, we can collect the three definitions for $\neg$, $\wedge$ and $\vee$ in a single extended truth table as follows:

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$P \vee Q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	F	F